

# Fast Two-Phase Topology Optimization of a Linear MEMS Gyroscope

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## Introduction

Microelectromechanical systems (MEMS) have subverted our everyday life as sensors and actuators components in consumer, medical or automotive applications. The design of many MEMS is determined by their resonant modes and respective frequency characteristics. However, the devices' resonance frequencies can often not be chosen independently. Furthermore, the geometric design alterations affect multiple resonance frequencies, simultaneously. This motivates a systematic approach for identifying designs with desired spectral properties.

A systematic approach to geometry alteration is provided by mathematical optimization. It alters the design based on a user-defined goal function and given constraints, which arise from e.g. the fabrication technology. Different optimization approaches give different levels of freedom to the optimizer, ranging from simple change of geometric dimensions in classical parameter optimization to topology optimization, that can evolve a cubic geometry into a torus. In this work, we present an approach, which combines an evolutionary algorithm with classical gradient-based topology optimization. The optimization is implemented in Python and we use PyAnsys [1] to build and solve the model. The potential of this combined approach for the design of MEMS is demonstrated on a linear gyroscope.

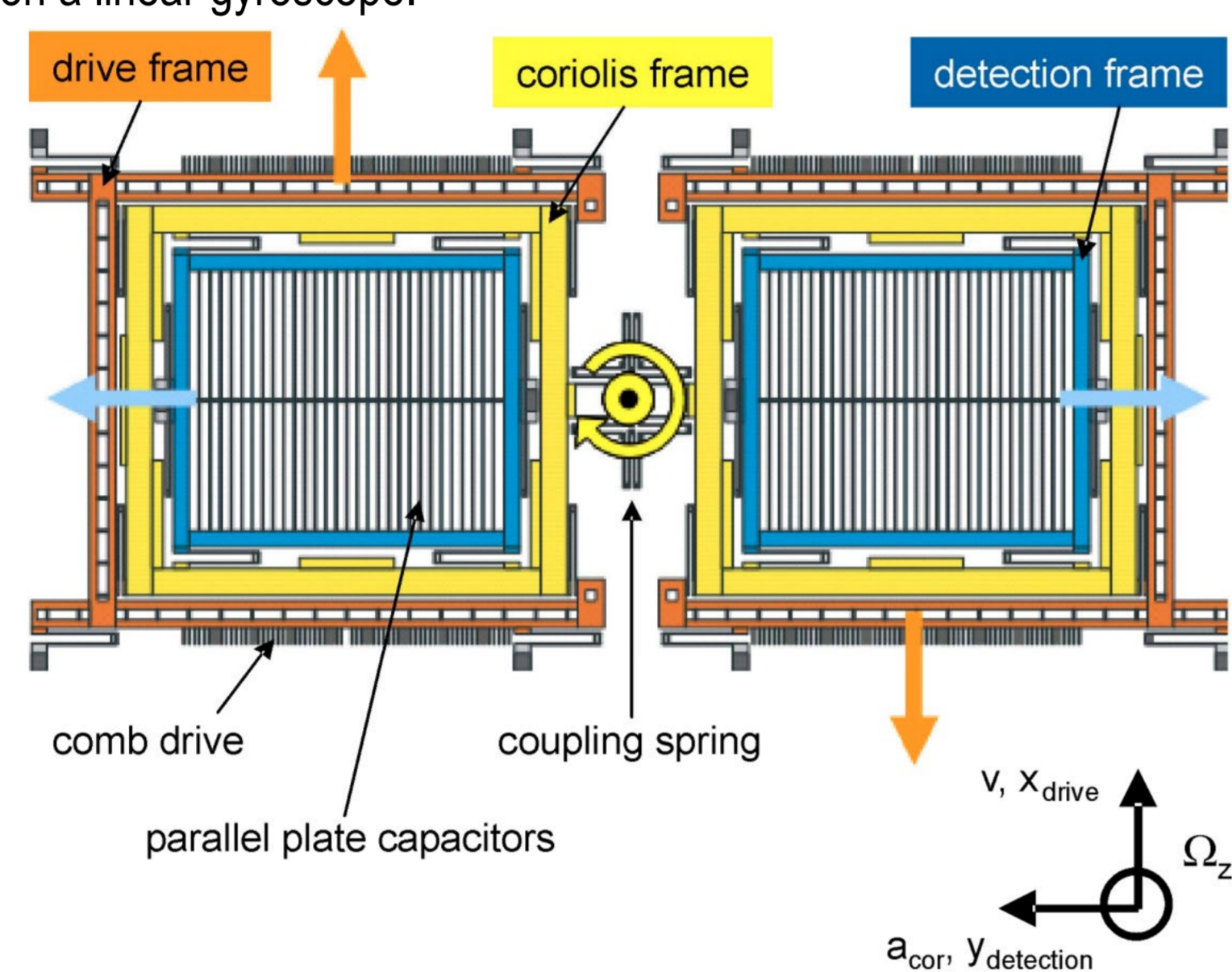


Figure 1. Schematic view of an anti-phase MEMS gyroscope. Drive motion in x-direction. Sensing motion in y-direction. The z-axis is the sensitive angular rate input axis [2].

## Methodology

Topology optimization problem:

$$\min_x \frac{1}{2} u_{\text{stat}}^T K u_{\text{stat}}$$

subject to:

$$\begin{aligned} (\omega_n - \omega_{n,\text{goal}})^2 - \epsilon^2 &\leq 0, \\ \frac{\omega_n^2}{\omega_{n,\text{goal}}^2} - \epsilon^2 &\leq 0, \\ (K - \omega_n^2 M) \Phi_n &= 0, \\ K u_{\text{stat}} &= F_{\text{stat}}, \\ \sum_e x_e v_e - V^* &= 0, \end{aligned}$$

In Phase 1, we use bi-directional evolutionary structural optimization (BESO) [3]. BESO is initiated with a completely filled design space. It successively eliminates elements with the lowest sensitivities with respect to the optimization objective and constraints. A predefined evolution rate determines how many elements are de-/activated in each iteration. A demonstration is shown in Figure 2. BESO works well with established finite-element software, which is a major advantage over classical approaches. These classical approaches are mostly based on varying elemental densities using simple isotropic material with penalization (SIMP) [4], i.e. where the elemental mass and stiffness matrices are scaled with the optimization variable  $x_e$  and a penalty factor  $p$ :

$$M_e(x_e) = x_e M_{e,\text{full}}, \quad K_e(x_e) = x_e^p K_{e,\text{full}}, \quad 0 < x_e \leq 1.$$

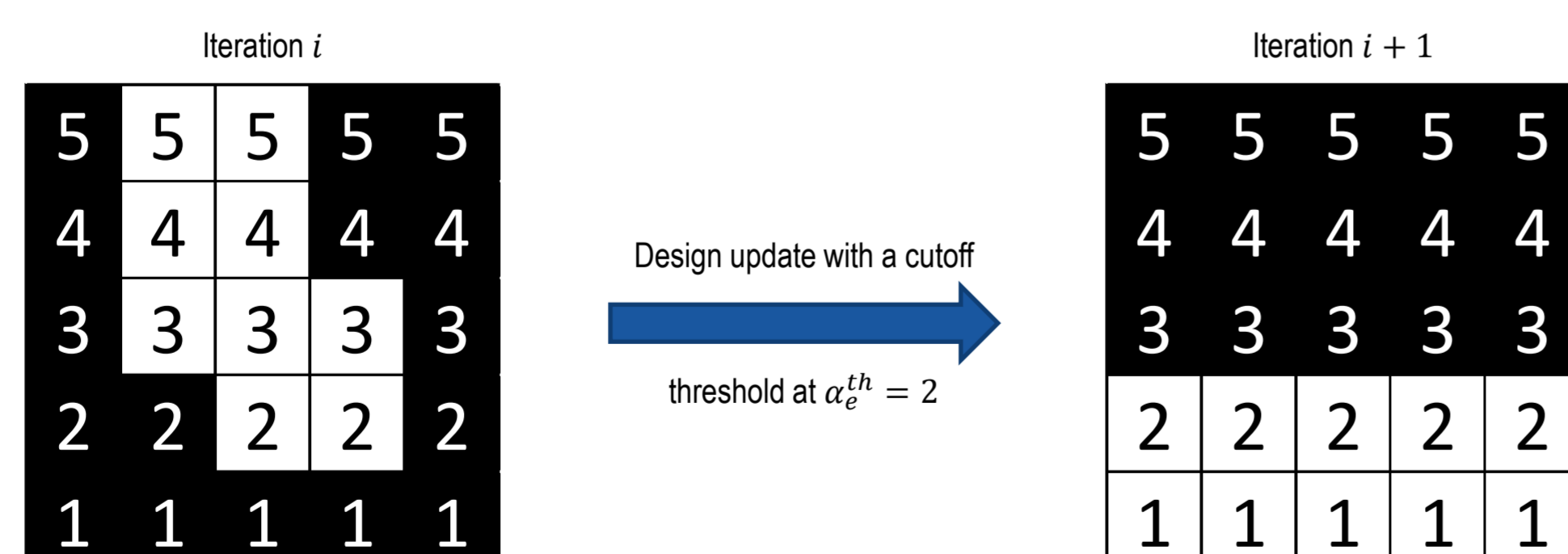


Figure 2: Diagram of the BESO update scheme. ■ represents active and □ deactivated elements. The numbers correspond to the elemental sensitivity values  $\alpha_e$ . Elements with sensitivities  $\alpha_e \leq \alpha_e^{\text{th}}$  are/remain deactivated and those above  $\alpha_e > \alpha_e^{\text{th}}$  are activated.

In Phase 2, we use SIMP and gradient-based continuous optimization to achieve desirable convergence behavior. In addition, mathematical model order reduction (MOR) is implemented in this phase for the reduction of computational effort, as we now have to solve the model equation in Python. In this work, we have implemented projective MOR using eigenbasis of the system consisting of its first 3 eigenvectors. A schematic of projective MOR is shown in Figure 3.

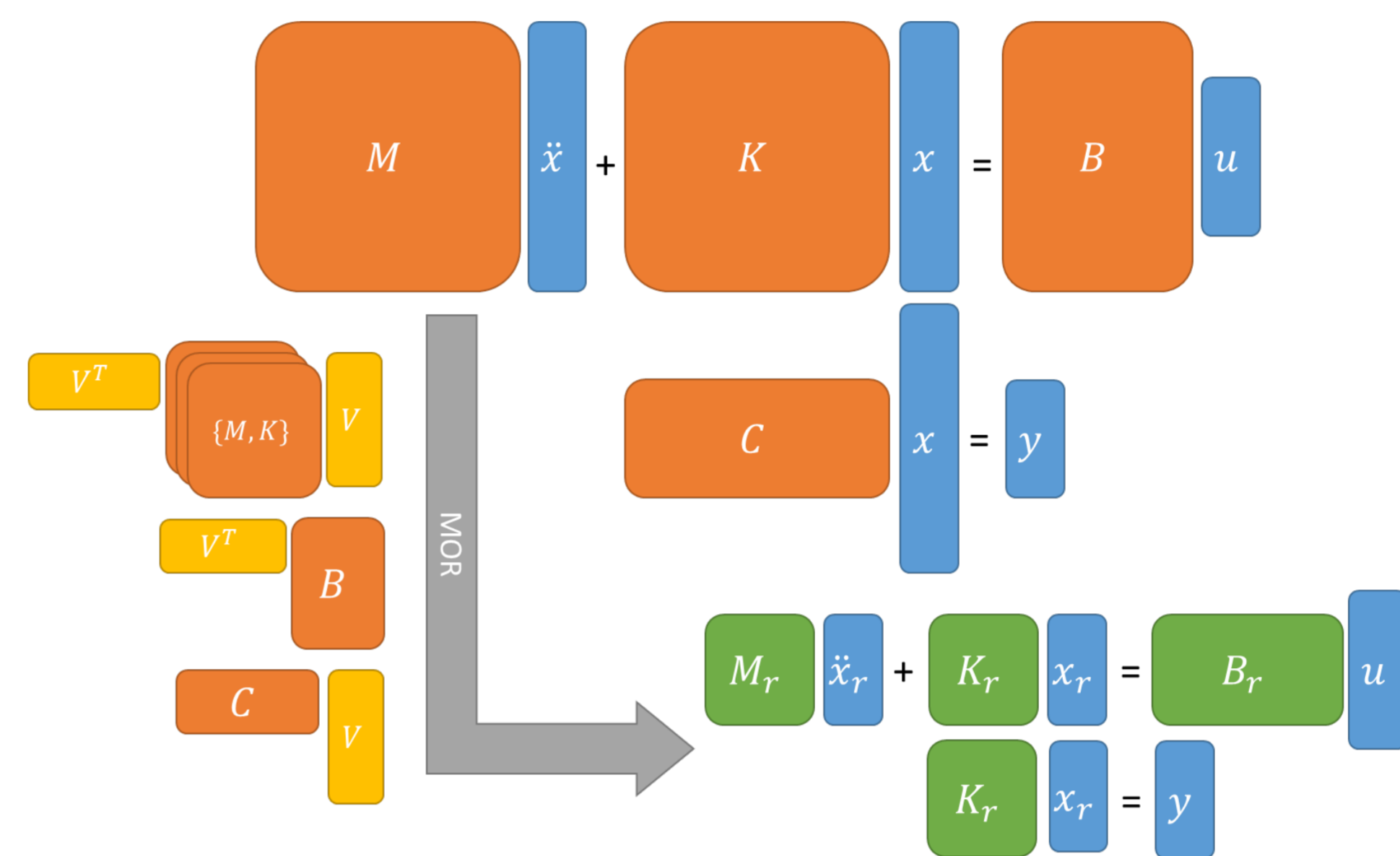


Figure 3: The system matrices  $M$ ,  $K$ ,  $B$  and  $C$  are projected onto a reduced eigenspace by multiplication with the projection matrix  $V$ , i.e. the columns of matrix  $V$  consist of the first three eigenvectors of the matrix pencil  $(K, M)$ . In this way a reduced order model is created, which is compact but highly accurate [5].

## Numerical Example

Gyroscopes are devices used for measuring angular acceleration. Their MEMS version are applied in consumer electronics, e.g., in smartphones or smartwatches. A MEMS gyroscope typically consists of a proof mass suspended by a compliant structure that allows the mass to oscillate in two orthogonal directions. These two movements correspond to the first two resonance modes of the structure, where the first mode is called the drive mode and the second the sense mode. The design goal of a vibrational gyroscope is to have the sense mode at 2-10% higher frequency than the drive mode, while the frequencies of higher (spurious) modes are significantly higher than the sense mode. For our optimization, we use a normalized material and set the goal frequencies to values shown in Table 1. The design space and optimized geometry are shown in Figure 4.

Table 1. Constraints for the TO (based on [6]) and the resonance frequencies of the final geometry.

Gyroscope	Objective + Constraints	Final Geometry
Volume fraction	0.5	0.5
Mode 1	$\omega_{1,\text{goal}} = 4 \text{ Hz}$	$\omega_1 = 3.92 \text{ Hz}$
Mode 2	$\omega_{2,\text{goal}} \geq 1.4\omega_1$	$\omega_2 = 5.27 \text{ Hz}$
Mode 3	$\omega_{3,\text{goal}} = 3.8 \text{ Hz}$	$\omega_3 = 3.84 \text{ Hz}$

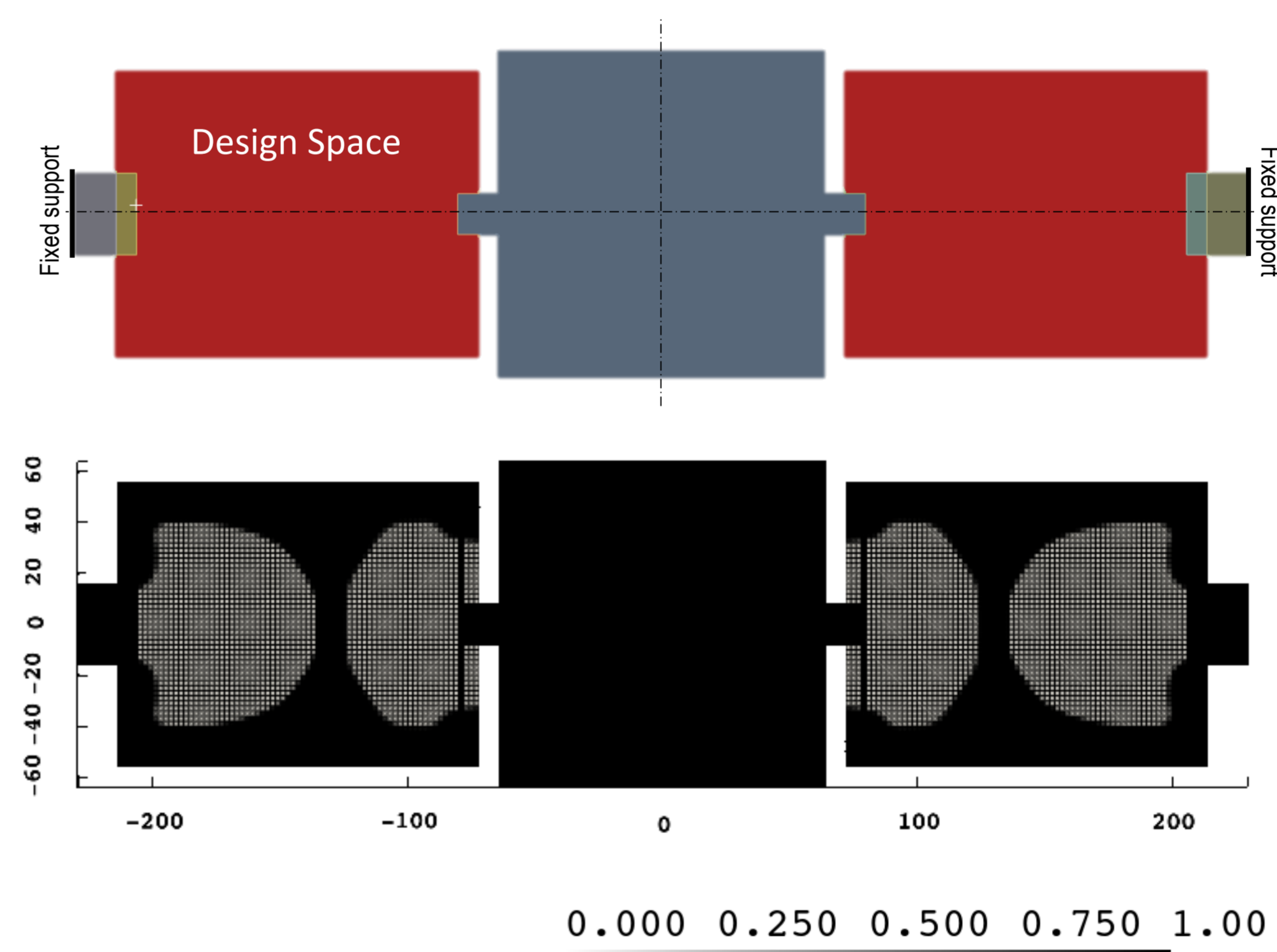


Figure 4: Design space of for the optimization of the gyroscope (top), which is chosen based on [6], and the corresponding optimal design (bottom). The gray areas are excluded from the optimization; red areas are subject to design alterations considering shown symmetry.

## Acknowledgement

This work is funded by German Research Foundation (DFG) – project no. 446995632 (optiMuM).



## References

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