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Nonlinear Model Order Reduction of a MEMS Actuator by a Trajectory Piecewise-Linear Approximation

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Introduction

Fast and reliable models enable microsystems to reach their full potential by facilitating an efficient design process and feedback-controlled operation. Models generated by the finite element method (FEM) are reliable but lack efficiency. Mathematical model order reduction (MOR) tackles this challenge by constructing highly accurate, but much more efficient surrogate models [1]. However, nonlinearities in the form of geometrical nonlinearities, electromechanical coupling, or mechanical contact remain challenging for MOR and require additional methods. These approaches rely on detailed mathematical descriptions, which are usually not accessible within industry-level software. The trajectory piecewise-linear (TPWL) approximation constitutes a robust alternative [2]. The key idea is to approximate the nonlinear system as a weighted sum of linearized ones obtained along a training trajectory. As this data is available in most industry-level FEM software, the approach enables nonlinear MOR of industry-relevant FEM models.
This work demonstrates the workflow of MOR and TPWL for a nonlinear micromechanical beam actuator. The actuator is developed within the "Kick and Catch"-project [3] and forms a part of an innovative microsystem presented in Fig. 1. The goal is a multistable quasistatic micromirror for large deflection angles. Omitting all structural connections to a moving body - in this case a spherical cap – enables the free motion. Consecutive contact events by four electrostatic beam actuators incrementally rotate the body as shown in Fig. 2.

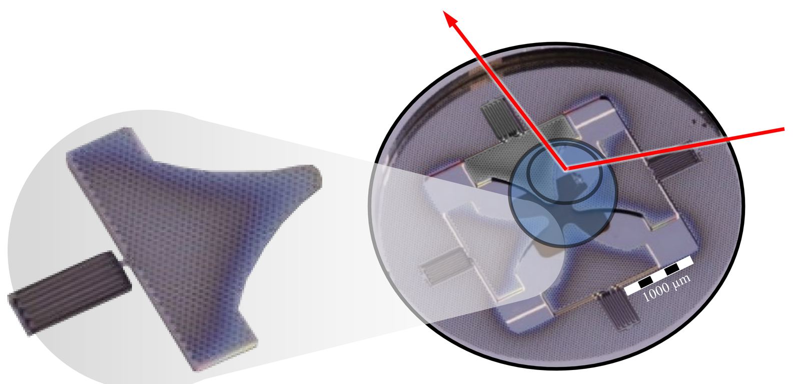

Fig. 1: Microscope picture of the Kick and Catch actuator system [3] emphasizing the electrostatic beam actuators. In addition, the spherical cap is included schematically, deflecting an incident light ray.

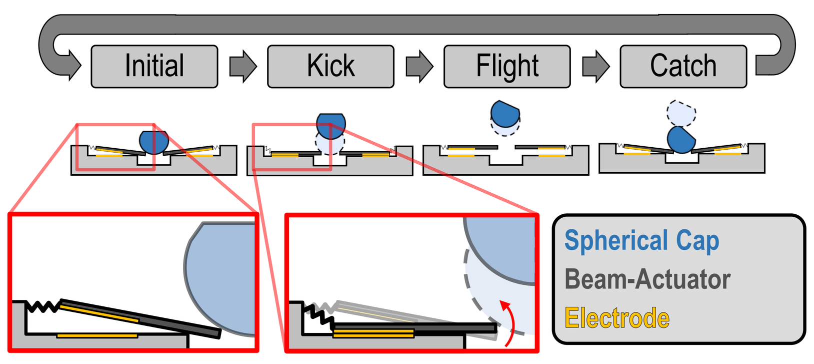

Fig. 2: Operating principle of the Kick and Catch microsystem [3]: a spherical cap rests on four electro-static beam actuators. Actuating the beams into pull-in transfers momentum to the spherical cap. Consecutive actuation achieves large rotation angles.

Methodology

For this case study, a single beam actuator is loaded with an out-of-plane downward force at its tip, causing mechanical contact to the supporting chip. A FEM model establishes the reference solution and provides the mathematical model for subsequent MOR. Projection-based MOR by proper orthogonal decomposition (POD) [4] creates a reduced order model that deploys TPWL to efficiently handle nonlinearities. Fig. 3 illustrates this workflow and Fig. 4 presents the load case and the idea of TPWL, which will be applied to the FEM model and its reduced version.

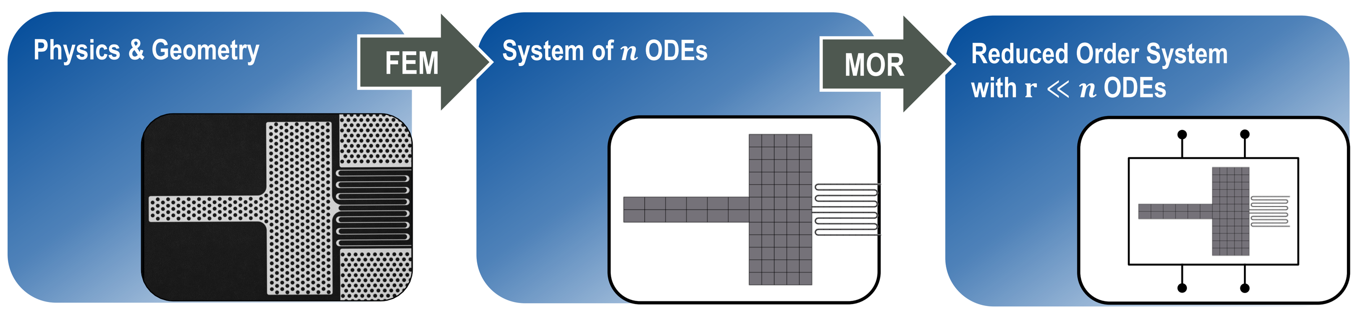

Fig. 3: Workflow of modeling: FEM models the beam actuator as a high-dimensional system of ordinary differential equations (ODEs). Subsequently, MOR constructs an accurate surrogate model of drastically smaller dimension.

Mathematically, the FEM model corresponds to a large-scale system of n=1602 nonlinear ordinary differential equations, here with nonlinear restoring forces $\boldsymbol{f}(\boldsymbol{x})$:

$$\Sigma\begin{cases}\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{E}\dot{\boldsymbol{x}} + \boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}\end{cases}, \quad \begin{array}{rr}\mathbf{M},\mathbf{E},\mathbf{K} \in \mathbb{R}^{nxn}, & \boldsymbol{x},\boldsymbol{f}(\boldsymbol{x}) \in \mathbb{R}^{n} \\ \boldsymbol{B} \in \mathbb{R}^{nxp}, & \boldsymbol{u} \in \mathbb{R}^{p} \\ \boldsymbol{C} \in \mathbb{R}^{qxn}, & \boldsymbol{y} \in \mathbb{R}^{q}\end{array}$$

MOR generates a surrogate model of much smaller dimension $r = 5$ by projecting the resulting equation onto an appropriate subspace spanned by the columns of matrix $\boldsymbol{V} \in \mathbb{R}^{nxr}$:

$$\Sigma_r\begin{cases}\underbrace{\boldsymbol{V}^T\boldsymbol{M}\boldsymbol{V}}_{\boldsymbol{M}_r}\ddot{\boldsymbol{x}}_r + \underbrace{\boldsymbol{V}^T\boldsymbol{E}\boldsymbol{V}}_{\boldsymbol{E}_r}\dot{\boldsymbol{x}}_r + \boldsymbol{V}^T\boldsymbol{f}(\boldsymbol{V}\boldsymbol{x}_r) = \underbrace{\boldsymbol{V}^T\boldsymbol{B}}_{\boldsymbol{B}_r}\boldsymbol{u} \\ \boldsymbol{y} = \underbrace{\boldsymbol{C}\boldsymbol{V}}_{\boldsymbol{C}_r}\boldsymbol{x}_r\end{cases}, \quad \begin{array}{rr}\mathbf{M_r},\mathbf{E_r},\mathbf{K_r} \in \mathbb{R}^{rxr}, & \boldsymbol{x} \in \mathbb{R}^{n} \\ \boldsymbol{B}_r \in \mathbb{R}^{rxp}, & \boldsymbol{u} \in \mathbb{R}^{p} \\ \boldsymbol{C}_r \in \mathbb{R}^{qxr}, & \boldsymbol{y} \in \mathbb{R}^{q}\end{array}$$

TPWL first linearizes the nonlinear restoring forces $\boldsymbol{f}(\boldsymbol{x})$ around N states $\boldsymbol{x}_i$ from the system's trajectory. A weighted sum of these linearizations forms a global approximation given by:

$$\boldsymbol{f}(\boldsymbol{x}(t)) \approx \sum_{i=1}^{N}\left(w_i(\boldsymbol{x})(-\boldsymbol{f}_i + \boldsymbol{K}_i \cdot \boldsymbol{x})\right)$$

This approximation is subsequently reduced and evaluated in terms of computational efficiency and accuracy by comparing it to the original FEM model's solution.

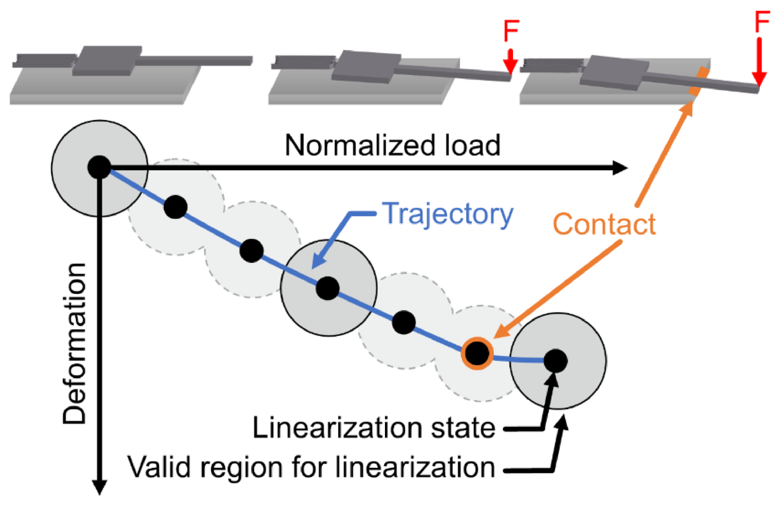

Fig. 4: The concept of the TPWL approximation: the load case illustrated in the top row relies on mechanical contact, rendering the model nonlinear. The TPWL approximation replaces the original system with a combination of linearized ones, which are obtained at different states along the trajectory. The linearized models' range of validity is indicated by circles at their linearization states.

Numerical Results

TPWL and its combination with MOR are applied to the FEM model in Fig. 3 to assess the methods' performances. The beam's vertical tip deflection constitutes the output quantity for comparison and error analysis. While the TPWL reduced order model is the one of interest, an additional TPWL-approximated original model allows for more nuanced conclusions.
Fig. 5 compares the beam's tip displacement computed by the FEM model, the TPWL model, and the TPWL reduced order model. In general, the relative error between the TPWL model and its reduced version is the lowest, identifying the TPWL approximation as the main source of deviation.
Fig. 6 lists the computational time for simulating each of the three models.

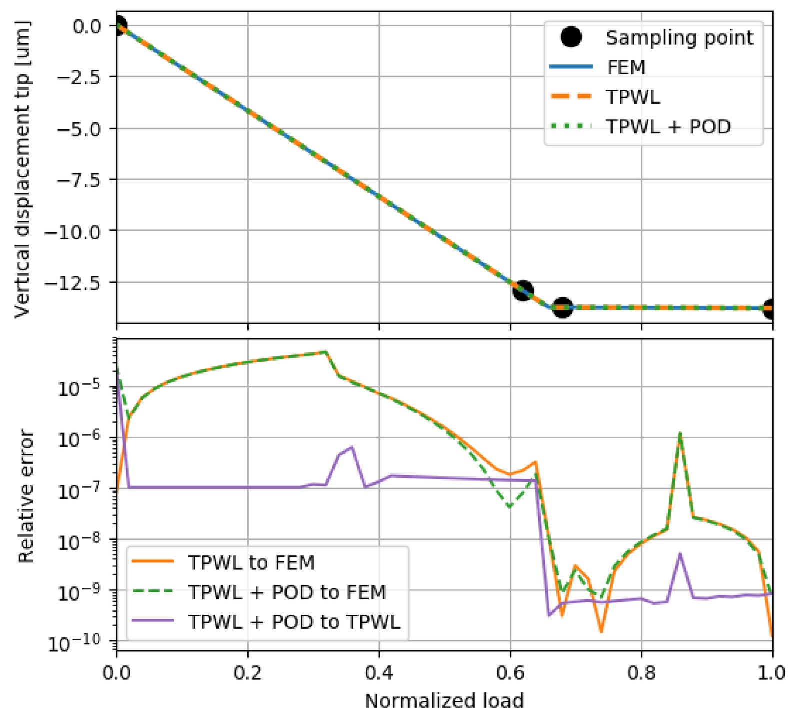

Fig. 5: Comparison of the beam's vertical tip displacement over the normalized load: The FEM solution constitutes the reference, which the TPWL model and its reduced version approximate. The plot also highlights sampling positions for linearized models utilized by the TPWL approximation. The bottom plot presents the relative errors between the three solutions to distinguish effects due to TPWL and due to POD. No relative error surpasses 10^{-4}, indicating excellent approximation quality.

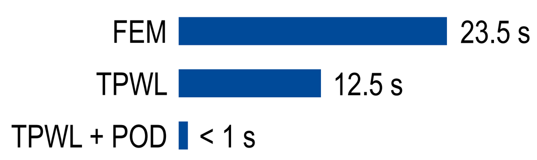

Fig. 6: Comparing the computational solution time between the original FEM model, the TPWL-approximated model, and its reduced version (Intel® Core™ i5-7600, 32 GB RAM).

Conclusion and Outlook

We successfully generated a highly efficient and accurate surrogate model of a nonlinear micro-actuator. The workflow relies exclusively on data available from industrial simulation software. However, the surrogate model is based on data obtained from training simulations and therefore, its prediction quality is limited to scenarios similar to the training input.
Future work might deploy this methodology to different models, extend the data acquisition by more sophisticated sampling schemes, or investigate different weighting procedures.

Acknowledgement

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References

[1] A. C. Antoulas, "Approximation of large-scale dynamical systems", Society for Industrial and Applied Mathematics: Philadelphia, 2005.
[2] M. Rewienski and J. White, "A trajectory piecewise-linear approach to model order reduction and fast simulation of nonlinear circuits and micromachined devices" IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, 2003.
[3] M. Farny and M. Hoffmann, "Kick & Catch: elektrostatisches Rotieren einer Kugel", MikroSystemTechnik Kongress, 2021.
[4] Kerschen, G.; Golinval, J.; Vakakis, A. F.; Bergman, L. A. The Method of Proper Orthogonal Decomposition for Dynamical Characterization and Order Reduction of Mechanical Systems: An Overview. Nonlinear Dyn 2005, 41, 147–169.

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