Fast Topology Optimization for Resonating Structures via Generalized Incremental Frequency Method and Modal Superposition-Based Model Order Reduction

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Abstract This contribution presents an approach to decrease the computational effort of the generalized incremental frequency method, initially used to obtain designs with minimal dynamic compliance under external harmonic excitation. This goal is achieved by utilizing modal superposition-based model order reduction technique. Furthermore, the method is adapted for topological design optimization of resonating structures, dealing with exact placement of resonance frequencies.

The generalized incremental frequency (GIF) method was introduced by Olhoff and Du in [1], in 2016. The method is based on changing material distribution to minimize the dynamical compliance of undamped elastic continuum structures for given external harmonic excitations. Different to the common design goal in such cases, which is to drive the structure's resonance frequencies as far away from the excitation frequency as possible, our goal is to have the resonant frequencies at locations of interest, e.g. the structure's resonance frequency should match the excitation frequency. A typical use case with such a design goal is vibration-based energy harvesting [2]. This goal is achieved by utilizing the property of the GIF method that steers the structure's resonance frequencies up or down by minimizing the dynamic compliance for a specific array of external driving frequencies.

As the GIF method requires an eigenfrequency analysis in each optimization loop, we use the eigenvectors obtained from this analysis as a basis for projection-based model order reduction, i.e. let the columns of the matrix $V \in \mathbb{R}^{n\times r}$ be the orthonormal eigenbasis corresponding to r eigenmodes of the structure, $K_d \in \mathbb{R}^{n\times n}$ be the dynamic stiffness matrix and P be the vector of amplitudes of external excitation, we can project the equilibrium equation $K_d U = P$ onto the subspace spanned by V as follows:

$$\underbrace{V^T K_d V}_{K_{d,r}} \underbrace{V^T U}_{U_r} = \underbrace{V^T P}_{P_r},$$

and solve the projected equilibrium equation $K_{d,r} U_r = P_r$ instead of the original equation. The approximation error when solving the projected equilibrium equation is negligible as long as the columns of V correspond to the dominant eigenmodes of the structure, which can be identified by different measures, e.g. [3]. The sensitivity can be computed as usual, either by also projecting the sensitivity K_d' onto the reduced order subspace or via the up-projection of the reduced order displacement vector U_r (i.e. $U \approx V U_r$).

As most structures have only a small number of dominant eigenmodes, $r \ll n$ applies. Therefore, by solving the lower dimensional projected equilibrium equation instead of the original equation, the computational effort can be significantly reduced in each optimization loop.

References

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