Binomische Formeln

I. $(a+b)^2 = a^2 + 2ab + b^2$

II. $(a-b)^2 = a^2 - 2ab + b^2$

III. $(a+b)(a-b) = a^2 - b^2$

pq - Formel

$$x^2 + px + q = 0$$

$$\Rightarrow \quad x_{1/2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

...und

$$\sqrt{x} \cdot \sqrt{x} = \sqrt{x \cdot x} = \sqrt{x^2} = x$$

Aufgabe 8.9. von Seite 178

a)
$$\begin{aligned} \sqrt{6x-11} + 6 &= 7 && / -6 \\ \sqrt{6x-11} &= 1 && / (\ )^2 \\ 6x - 11 &= 1 && / +11 \\ 6x &= 12 && / :6 \\ \underline{\underline{x}} &\underline{\underline{= 2}} \end{aligned}$$

b)
$$\begin{aligned} 5 - 3\sqrt{x+6} &= 2 && / -5 \\ -3\sqrt{x+6} &= -3 && / :(-3) \\ \sqrt{x+6} &= 1 && / (\ )^2 \\ x + 6 &= 1 && / -6 \\ \underline{\underline{x}} &\underline{\underline{= -5}} \end{aligned}$$

c) $\sqrt{x+m} - \sqrt{x-m} = \frac{x+m-n}{\sqrt{x+m}} \quad / \cdot \sqrt{x+m}$

$\sqrt{x+m} \cdot \sqrt{x+m} - \sqrt{x+m} \cdot \sqrt{x+m} = x+m-n$

$x+m - \sqrt{(x+m)(x-m)} = x+m-n \quad / -x-m$

$-\sqrt{(x+m)(x-m)} = -n \quad / :(-1)$

$\sqrt{x^2-m^2} = n \quad / (\ )^2$

$x^2 - m^2 = n^2 \quad / +m^2$

$x^2 = m^2+n^2 \quad / \sqrt{\ }$

$$\underline{\underline{x = \sqrt{m^2+n^2}}}$$

d) $\sqrt{2x+10} - \sqrt{4x-8} = 2 \quad / (\ )^2$

$(\sqrt{2x+10} - \sqrt{4x-8})^2 = 2^2$

$2x+10 - 2\sqrt{2x+10} \cdot \sqrt{4x-8} + 4x-8 = 4 \quad / -6x-2$

$-2\sqrt{(2x+10)(4x-8)} = 2-6x \quad / :(-2)$

$\sqrt{8x^2-16x+40x-80} = -1+3x \quad / (\ )^2$

$8x^2+24x-80 = (3x-1)^2$

$8x^2+24x-80 = 9x^2-2\cdot(3x)\cdot 1+1$

In die Normalform der Quadratischen Gleichung bringen und nach pq-Formel lösen

$8x^2+24x-80 = 9x^2-6x+1 \quad / -8x^2-24x+80$

$x^2 - 30x + 81 = 0$

$x_{1,2} = \frac{30}{2} \pm \sqrt{\left(\frac{30}{2}\right)^2 - 81}$

$= 15 \pm \sqrt{15^2-81}$

$= 15 \pm \sqrt{225-81}$

$= 15 \pm \sqrt{144}$

$= 15 \pm 12$

$x_1 = 27 \qquad x_2 = 3$

Die Probe muss zeigen welches Ergebnis korrekt ist.

d) Probe:

$x_1 = 27$ $\quad \sqrt{2 \cdot 27 + 10} - \sqrt{4 \cdot 27 - 8} \neq 2$

$$\sqrt{64} - \sqrt{100} \neq 2$$

$$8 - 10 \neq 2$$

$$-2 \neq 2$$

$x_2 = 3$ $\quad \sqrt{2 \cdot 3 + 10} - \sqrt{4 \cdot 3 - 8} = 2$

$$\sqrt{16} - \sqrt{4} = 2$$

$$4 - 2 = 2$$

$$2 = 2 \quad \text{q.e.d.}$$

Lösung: $\underline{\underline{x = 3}}$

e) $\sqrt{2x+5} = 2x - 1 \quad /(\ )^2$

$$2x + 5 = (2x-1)^2$$

$$2x + 5 = 4x^2 - 2 \cdot 2x + 1$$

$$2x + 5 = 4x^2 - 4x + 1 \quad /-2x-5$$

$$4x^2 - 6x - 4 = 0 \quad /:4$$

$$x^2 - \frac{6}{4}x - 1 = 0$$

$$x^2 - 3/2\,x - 1 = 0$$

$$x_{1,2} = \frac{3}{4} \pm \sqrt{\left(\frac{3}{4}\right)^2 + 1}$$

$$= 3/4 \pm \sqrt{\frac{9}{16} + \frac{16}{16}} = \frac{3}{4} \pm \sqrt{\frac{25}{16}}$$

$$= \frac{3}{4} \pm \frac{5}{4}$$

$$x_1 = \frac{8}{4} = 2$$

$$x_2 = -\frac{2}{4} = -\frac{1}{2}$$

e) Probe

$x_1 = 2$	$x_2 = -1/2$
$\sqrt{2\cdot 2+5} = 2\cdot 2-1$	$\sqrt{2\cdot(-1/2)+5} = 2\cdot(-1/2)-1$
$\sqrt{9} = 3$	$\sqrt{4} = -2$
$3 = 3$	$2 = -2$ f
$x=2$ ist eine Lösung der Gleichung $\sqrt{2x+5} = 2x-1$	$2 \neq -2$ daher $x = -1/2$ keine Lösung

$$\underline{\underline{x = 2}}$$

f) $\sqrt{3x-3} - \sqrt{x-3} = \sqrt{2x-4} \quad /(\ )^2$

$$(3x-3) - 2\cdot\sqrt{x-3}\cdot\sqrt{3x-3} + (x-3) = 2x-4 \quad /-4x+6$$

$$-2\sqrt{3x-3}\cdot\sqrt{x-3} = -2x+2 \quad /:(-2)$$

$$\sqrt{3x-3}\cdot\sqrt{x-3} = x-1 \quad /(\ )^2$$

$$(3x-3)\cdot(x-3) = (x-1)^2$$

$$3x^2-3x-9x+9 = x^2-2x+1 \quad /-x^2+2x-1$$

$$2x^2-10+8 = 0 \quad /:2$$

$$x^2-5+4 = 0$$

$$x_{1,2} = \frac{5}{2} \mp \sqrt{\frac{25}{4}-\frac{16}{4}} = \frac{5}{2} \pm \sqrt{\frac{9}{4}} = \frac{5}{2} \pm \frac{3}{2}$$

$$x_1 = \frac{8}{2} = 4 \qquad x_2 = \frac{2}{2} = 1$$

Probe

$x_1=4$ $\sqrt{12-3} - \sqrt{4-3} = \sqrt{8-4}$

$3 - 1 = 2$ ✓ $\quad \underline{\underline{x_1 = 4}}$

$x_2=1$ $\sqrt{3-3} - \sqrt{1-3} = \sqrt{2-4}$

$\sqrt{3-3} = \sqrt{0} = 0$

$$\left.\begin{array}{l} 1-3 < 0 \\ 2-4 < 0 \end{array}\right\} \text{nicht im Definitionsbereich}$$

g) $\sqrt{x+3} + \sqrt{2x-8} = \frac{15}{\sqrt{x+3}} \quad / \cdot \sqrt{x+3}$

$x+3 + \sqrt{(2x-8)(x+3)} = 15 \quad / -x-3$

$\sqrt{2x^2-8x+6x-24} = 12-x \quad / (\ )^2$

$2x^2-2x-24 = 144-24x+x^2$

$x^2+22x-168 = 0$

$x_{1,2} = -11 \pm \sqrt{121+168} = -11 \pm \sqrt{289}$

$= -11 \pm 17$

$\underline{\underline{x_1 = 6}}$ ✓ $\quad x_2 = -28$ f

Definitionsbereich

$x+3 \geq 0 \quad \wedge \quad 2x-8 \geq 0$

$x \geq -3 \quad \wedge \quad x \geq 4$

$\mathbb{D} = \{x \in \mathbb{R};\ x \geq 4\}$

h) $\sqrt{x+1+\sqrt{3x+4}} = 3 \quad / (\ )^2$

$x+1+\sqrt{3x+4} = 9 \quad / -x-1$

$\sqrt{3x+4} = 8-x \quad / (\ )^2$

$3x+4 = 64-16x+x^2 \quad / -3x-4$

$x^2-19x+60 = 0$

$x_{1,2} = \frac{19}{2} \pm \sqrt{\left(\frac{19}{2}\right)^2 - 60} = \frac{19}{2} \pm \sqrt{\frac{361}{4} - \frac{240}{4}} = \frac{19}{2} \pm \sqrt{\frac{121}{4}}$

$= 19/2 \pm 11/2$

$x_1 = 30/2 = 15 \quad \underline{\underline{x_2 = 8/2 = 4}}$

Probe

x_1: $\sqrt{16+\sqrt{49}} = 3$

$\sqrt{16+7} = 3$

$\sqrt{23} = 3$ f

x_2: $\sqrt{5+\sqrt{16}} = 3$

$\sqrt{5+4} = 3$

$\sqrt{9} = 3$

$3 = 3$ q.e.d.

i) $\sqrt{5x^2-3x-4} = 2x \quad /(\ )^2$

$5x^2-3x-4 = 4x^2 \quad / -4x^2$

$x^2-3x-4 = 0$

$x_{1,2} = 3/2 \pm\sqrt{9/4 + 16/4} = \frac{3}{2} \pm\sqrt{\frac{25}{4}} = 3/2 \pm 5/2$

$x_1 = \frac{8}{2} = 4 \qquad x_2 = -\frac{2}{2} = -2$

Probe

$\sqrt{5\cdot 4^2 - 3\cdot 4 - 4} = 2\cdot 4$

$\sqrt{80 - 12 - 4} = 8$

$\sqrt{64} = 8 \rightarrow \underline{\underline{x=4}}$

$\sqrt{5(-1)^2 - 3(-1) - 4} = 2\cdot(-1)$

$\sqrt{5 + 3 - 4} = -2$

$\sqrt{4} = -2 \rightarrow \underline{\underline{x=-2}}$

k) $\sqrt{2x-1} + \sqrt{3x+10} = \sqrt{11x+9} \quad /(\ )^2$

$2x-1 + 2\sqrt{2x-1}\sqrt{3x+10} + 3x + 10 = 11x+9 \quad / -5x-9$

$2\sqrt{(2x-1)(3x+10)} = 6x \quad /:2$

$\sqrt{6x^2+20x-3x-10} = 3x \quad /(\ )^2$

$6x^2 + 17x - 10 = 9x^2 \quad / -9x^2$

$-3x^2 + 17x - 10 = 0 \quad /:(-3)$

$x^2 - \frac{17}{3}x + \frac{10}{3} = 0$

$x_{1,2} = \frac{17}{6} \pm\sqrt{\left(\frac{17}{6}\right)^2 - \frac{10}{3}} = \frac{17}{6} \pm\sqrt{\frac{289}{36} - \frac{120}{36}}$

$= \frac{17}{6} \pm\sqrt{\frac{169}{36}} = \frac{17}{6} \pm \frac{13}{6}$

$\underline{\underline{x_1 = \frac{30}{6} = 5}} \qquad \underline{\underline{x_2 = \frac{4}{6} = \frac{2}{3}}}$

e) $\sqrt{x+1} + \sqrt{3x-5} - \sqrt{x-2} - \sqrt{3x} = 0 \quad / + \sqrt{x-2} + \sqrt{3x}$

$$\sqrt{x+1} + \sqrt{3x-5} = \sqrt{x-2} + \sqrt{3x} \quad / (\ )^2$$

$$(x+1) + 2\sqrt{(x+1)(3x-5)} + (3x-5) = (x-2) + 2\sqrt{(x-2)3x} + 3x \quad / -4x+2-2\sqrt{(x+1)(3x-5)}$$

$$-2 = 2\left[\sqrt{(x-2)3x} - \sqrt{(x+1)(3x-5)}\right] \quad / :2$$

$$\sqrt{3x^2-6x} - \sqrt{3x^2+3x-5x-5} = -1 \quad / (\ )^2$$

$$(3x^2-6x) - 2\sqrt{(3x^2-6x)(3x^2-2x-5)} + (3x^2-2x-5) = 1 \quad / -6x^2+8x+5$$

$$-2\sqrt{9x^4-6x^3-15x^2-18x^3+12x^2+30x} = -6x^2+8x+6 \quad / :(-2)$$

$$\sqrt{9x^4-24x^3-3x^2+30x} = 3x^2-4x-3 \quad / (\ )^2$$

$$9x^4-24x^3-3x^2+30x = (3x^2-4x-3)(3x^2-4x-3)$$

$$9x^4-24x^3-3x^2+30x = 9x^4-12x^3-9x^2-12x^3+16x^2+12x-9x^2+12x+9$$

$$9x^4-24x^3-3x^2+30x = 9x^4-24x^3-2x^2+24x+9 \quad / -9x^4+24x^3+2x^2-24x-9$$

$$-x^2+6x-9 = 0 \quad / :(-1)$$

$$x^2-6x+9 = 0$$

$$x_{1,2} = \frac{6}{2} \mp \sqrt{\left(\frac{6}{2}\right)^2 - 9} = 3 \pm 0$$

$$\underline{\underline{X = 3}}$$