Compact Thermal Model of Human Tissue

Chengdong Yuan1,2, Gunasheela Sadashiviah1, Evgenii B. Rudnyi3, Tamara Bechtold1,2
1 Institute for Electronic Appliances and Circuits | University of Rostock; 2 Department of Engineering, Jade University of Applied Sciences; 3 CADFEM GmbH

Full-Scale Thermal Model:
The heat conduction in the tissue can be described by the bio-heat equation of Pennes [2]. After the spatial discretization with finite element method the model reads:

\[
\sum_i \left( \frac{E \cdot T_i(t) + A \cdot T_i(t) - B \cdot u(T_i(t)) - \rho_b c_b (T_i(t) - T_{amb}) + Q_{is}}{\gamma_i} \right) = C_i \cdot T_i(t)
\]

where \( T \in \mathbb{R}^N \) is the vector of unknown temperatures and \( Q_{is} \) and \( Q_{is} \) are the temperature-dependent perfusion and constant metabolic heat generation rates. \( T_{amb} = 37 \degree C \) is the arterial blood temperature. The heat dissipated from the skin surface is modelled by convection boundary condition \( q_r = h \cdot (T(t) - T_{amb}) \), where \( q_r \) is the heat flux normal to the boundary skin surface, \( T_{amb} \) is the ambient temperature and \( h \) is the film coefficient.

Reduced-Order Thermal Model:
In system (1), the non-linearity occurs in the input vector, whereas all other system matrices are constant. After applying the block-Arnoldi algorithm [3] to the system (1), we obtain the reduced model with dimension \( r \ll N \):

\[
\sum_i \frac{V_i^T \cdot z(t) + V_i^T \cdot z(t) - V_i^T \cdot B \cdot u(z(t)) - V_i^T \cdot A \cdot T_i(t)}{\gamma_i} = C_i \cdot z(t)
\]

In (2) the time-dependent temperature vector is sufficiently described in low-order subspace \( V \) with \( T(t) = V \cdot z(t) \), where \( z(t) \) is the reduced state vector.

Model Order Reduction of Nonlinear-Input Model:
In the first approach [4], we study a simplified cubic human tissue model (Fig. 2 left) and account for the nonlinear source term at the system-level. We segment the geometry into \( i \) segments and approximate the blood perfusion heat generation in each segment as constant:

\[
Q_i = \rho_b c_b (T_i(t) - T_{amb}) + Q_{is}/i
\]

where \( T_{amb} \) denotes the average temperature in each segment, which is back-coupled to the corresponding input at the system-level (Fig. 2 right).

In the second approach [5], a linearized element heat generation vector \( \tilde{q} \) is obtained by weighted load-vector snapshots at \( i \) points in time \( t_j \):

\[
\tilde{q} = \sum_j w_j \cdot \tilde{q}_j
\]

where \( w_j \) weights the vectors and \( \tilde{q}_j \) denotes the element heat generation vector at time \( t_j \). As approach (4) is only applicable to simple geometries and approach (5) will only work for the step response, in this work, we suggest to transfer the temperature-dependent heat generation rate to the left-hand-side and integrate it in the global heat conductivity matrix:

\[
\sum_i \left( E \cdot T_i(t) + A \cdot T_i(t) - B \cdot u(T_i(t)) - \rho_b c_b (T_i(t) - T_{amb}) + Q_{is} \right) \frac{1}{\gamma_i} = C_i \cdot T_i(t)
\]

where \( l \in \mathbb{R}^{r \times N} \) is a unity matrix if perfusion takes place in all volumes of the model. In our application, \( \sum_i \) is the extension of Model Reduction inside ANSYS® [6], we use the analogy between the \( q_{is} \) and \( q_{is} \) to treat blood perfusion heat generation as a \( "convection-type" \) effect. This method has been tested on the realistic forearm model with 104,295 degrees of freedom (DoF) (Fig. 3). We were able to generate highly accurate and compact reduced order model with only 30 DoF (Fig. 4). This approach will be used to find an optimal position of the TEG inside the human body.

References