

Compact Thermal Model of Human Tissue

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Introduction

An ever increasing aging of the European populations necessitates the development of advanced regenerative therapies. Electrically active implants support tissue generation through electrical stimulation. One of the major drawbacks of current implants is their limited lifetime, which requires repeated surgeries to replace the depleted unit. Harvesting the mechanical or thermal energy of the human body and converting it into electrical energy is a strategy to improve the implants lifetime up to full autonomy.

Model Order Reduction Methodology

Full-Scale Thermal Model:

The heat conduction in the tissue can be described by the bio-heat equation of Pennes [2]. After the spatial discretization with finite element method the model reads:

In this work, thermoelectric generators (TEGs), which use the Seebeck effect to transform the body's thermal energy into electrical voltage, are investigated as supplementary power sources for electrically active implants.



Fig. 1: Left: TEG with cylindrical housing and thermocouple legs array inside; Right: TEG-powered implant inside tissue [1].

Here, the TEG consists of two metallic discs and an array of thermocouple legs (Fig. 1 left). We propose to position the TEG inside the fat layer where the maximal temperature gradient is expected (Fig. 1 right). To efficiently find the optimal position for the TEG within a human body, we aim for creating a compact thermal body model via mathematical methods of model order reduction (MOR).

$$\sum_{N} \begin{cases} E \cdot \dot{T}(t) + A \cdot T(t) = \underbrace{B \cdot u(T(t))}_{Q} = \underbrace{\rho_b c_b \omega(T_a - T(t))}_{Q_p} + Q_m \\ y(t) = C \cdot T(t) \end{cases}$$
(1)

where $T(t) \in \mathbb{R}^N$ is the vector of unknown temperatures and Q_p and Q_m are the temperaturedependent perfusion and constant metabolic heat generation rates. $T_a = 37 \text{ °C}$ is the arterial blood temperature. The heat dissipated from the skin surface is modelled by convection boundary condition $q_{\perp} = h \cdot (T(t) - T_{ambient})$, where q_{\perp} is the heat flux normal to the boundary skin surface, $T_{ambient}$ is the ambient temperature and h is the film coefficient.

Reduced-Order Thermal Model:

In system (1), the non-linearity occurs in the input vector, whereas all other system matrices are constant. After applying the block-Arnoldi algorithm [3] to the system (1), we obtain the reduced model with dimension $r \ll N$:

$$\Sigma_{r} \begin{cases} \underbrace{V^{T} E V}_{E_{r}} \cdot \dot{z}(t) + \underbrace{V^{T} A V}_{A_{r}} \cdot z(t) = \underbrace{V^{T} B}_{B_{r}} \cdot u(z(t)) \\ y(t) = \underbrace{C V}_{C_{r}} \cdot z(t) \end{cases}$$
(2)

In (2) the time-dependent temperature vector is sufficiently described in low-order subspace subspace V with $T(t) \approx V \cdot z(t)$, where z(t) is the reduced state vector.

Simulation Results

Model Order Reduction of Nonlinear-Input Model:

In the first approach [4], we study a simplified cubic human tissue model (Fig. 2 left) and account for the nonlinear source term at the system-level. We segment the geometry into *i* segments and approximate the blood perfusion heat generation in each segment as constant:



$$Q_i = \rho_b c_b \omega \left(T_a - T_{avg_i} \right) + Q_m / i \tag{3}$$

where T_{avg_i} denotes the average temperature in each segment, which is back-coupled to the corresponding input at the system-level (Fig. 2 right).



In the second approach [5], a linearized elemental heat generation vector \overline{Q} is obtained by weighted load-vector snapshots at s points in time (t_i) :

$$\bar{Q} = \sum_{j=1}^{s} w_j \, q_j \tag{4}$$

where w_i denote the weights and q_i denote the elemental heat generation vector at time t_i .

As approach [4] is only applicable to simple geometries and approach [5] will only work for the step response, in this work, we suggest to transfer the temperature-dependent heat generation rate to the left-hand-side and integrate it in the global heat conductivity matrix:

Fig. 3: Left: Human left-hand forearm model with bones and blood vessels adapted from [7]; Right: Temperature distribution as obtained from steady-state thermal analysis.



Fig.4: Left: Temperature comparison between full model with 104,295 DoF and reduced model with 30 DoF in muscle, fat and skin tissue of the left-hand forearm model (at selected nodes); Right: Error between the full and reduced model.

References

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$\widetilde{\Sigma_N} \begin{cases} E \cdot \dot{T}(t) + (A + \rho_b c_b \omega \cdot I) \cdot T(t) = \rho_b c_b \omega T_a + Q_m \\ y(t) = C \cdot T(t) \end{cases}$ (5)

where $I \in \mathbb{R}^{N \times N}$ is a unity matrix if perfusion takes place in all volumes of the model. In our implementation, which is an extension of Model Reduction inside ANSYS[®] [6], we use the analogy between the Q_p and q_{\perp} to treat blood perfusion heat generation as a "convection-type" effect. This method has been tested on the realistic forearm model with 104,295 degrees of freedom (DoF) (Fig. 3). We were able to generate highly accurate and compact reduced order model with solely 30 DoF (Fig. 4). This approach will be used to find an optimal position of the TEG inside the human body.

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