

## Design Optimization of Multi-Physical and Multi-Resonant Microsystems Using Topology Optimization and Model Order Reduction

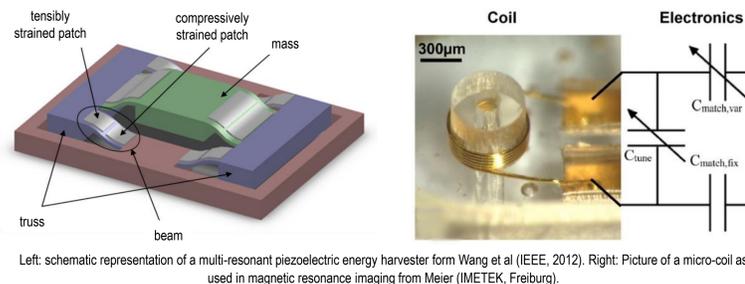
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### Introduction

Some say, "the soul of beings is their scent". While this statement is a controversial one, few would argue that the "soul" of every technical system indeed comes from their shape. As the shape of a violin define whether it is a Stradivarius or a plaything, the shape of a microsystem significantly influences its performance. In this project, we are looking for a fast and automated approach for optimal shape design of multi-physical and multi-resonant microsystems using two advanced mathematical approaches: topology optimization and model order reduction.

### Design Goal of Multi-Physical and Multi-Resonant Microsystems

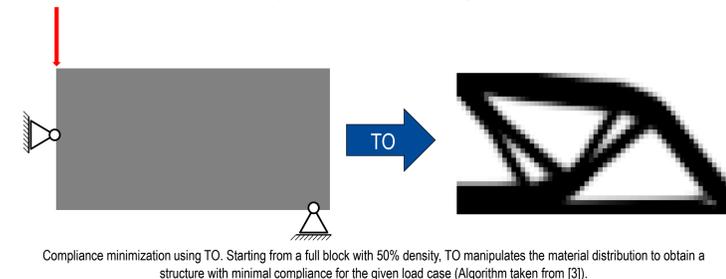
Nowadays, micro-electro-mechanical systems (MEMS) involve multiple physical domains. Some use resonance effects, for e.g. motion detection or energy harvesting. Some even include multiple subsystems operating at different resonant frequencies and are referred to as multi-resonant. Everyday-life examples are e.g. gyroscopes in smartphones, laser scanners in head-up displays or magnetic resonance detectors. Design of these devices aims at specific placing of system's eigenfrequencies. For some applications, the eigenfrequencies need to be significantly different to minimize energy transfer between the subsystems, while for others, the coupling is an integral part for system's functionality. In addition the eigenmodes have to be excitable (and thereby practically usable) and the mode shapes have to exhibit desirable deformations with a required amplitude within the spatial region of interest. Finally, these characteristics have to be preserved under varying temperature and pressure conditions.



### Topology Optimization (TO)

The methodology of TO was firstly introduced in the late 1980s. However, this methodology only starts getting appreciation by industry and academia in recent years, when 3D printing and other additive manufacturing methods have become popular. That is because TO can fully take advantage of the near to non-existent limitation of these manufacturing methods: Compare to conventional parameter optimization, TO does not rely on a given initial design based on an experienced designer's intuition but search for a optimal shape from the scratch, which often leads to revolutionary designs.

TO manipulates material distribution to optimize a certain characteristic of the system, i.e. to minimize compliance or to optimally distribute a systems eigenvalues.



Obviously, the detail level of fine structure depends on the size of material blocks. Therefore, one can imagine that for more advanced structures, where a fine level of detail is required, e.g. in the domain of microsystems, the computational effort is the bottleneck of the TO methodology.

### Model Order Reduction (MOR)

As technical system like MS has become increasingly complex, same applies to their mathematical description. Thus, simulation of these systems has become time consuming. The mathematical framework of MOR has been introduced tackle this issue and has rapidly become state-of-the-art in today's industry. This goal is achieved by replacing the original high dimensional system model with a significant lower level surrogate while preserving input/output behavior and other properties of the system, e.g. passivity or stability.

Modern MOR methods are projection-based. The basic idea assumes that the trajectory of a dynamical system will not deploy all parts of the state space equally often, but be mainly constrained to a significantly lower dimensional subspace. Accordingly, the system matrices are projected into exactly these subspaces.

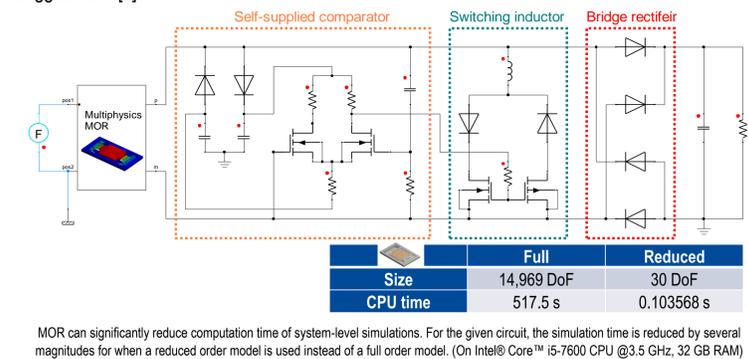
$$\begin{aligned} E \dot{x} &= A x + b u \\ y &= c x + d u \end{aligned} \quad x \in \mathbb{R}^n$$

$$\xrightarrow{\text{MOR}} \begin{aligned} E_r \dot{x}_r &= A_r x_r + b_r u \\ y_r &= c_r x_r + d u \end{aligned} \quad x_r \in \mathbb{R}^q, q \ll n$$

$$\begin{aligned} E_r &= W^T E V \\ A_r &= W^T A V \\ b_r &= W^T b \\ c_r &= c V \end{aligned}$$

The system matrices are projected into suitable and significantly lower dimensional subspaces by some projection matrices V and W.

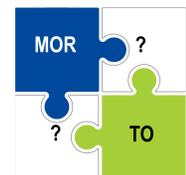
There are different approaches to obtain suitable projection matrices and thus subspaces. The two well-known and most used approaches are Krylov subspace methods and modal superposition. One may encounter stability issues when reducing microsystems. Countermeasures are e.g. suggested in [4].



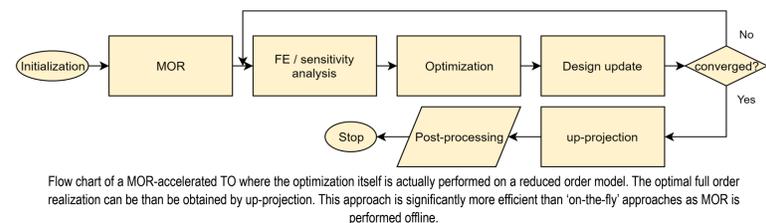
MOR can significantly reduce computation time of system-level simulations. For the given circuit, the simulation time is reduced by several magnitudes for when a reduced order model is used instead of a full order model. (On Intel® Core™ i5-7600 CPU @3.5 GHz, 32 GB RAM)

### The Missing Link

In the last few years, several approaches to accelerate TO via MOR has been proposed. They all involve a so-called 'on-the-fly' approach, which solely utilizes MOR to reduce the computation time of function evaluation during the TO loops. The extra effort of MOR in each loop is minimized by recycling stored data of preceding computations, e.g. reuse the reduced order model (ROM) of the system for several optimization loops as long as the error of the ROM does not exceed a predefined tolerance. The problem with this approach, however, is that increasing number of topology changes, the ROM has to be re-computed more often. As MOR itself has comparable computational effort to modal decomposition of the system, the advantage of including MOR becomes insignificant.



The goal of this project is to explore a more convenient approach to marry TO with MOR, where MOR is only performed offline. TO is then only performed in a lower dimensional subspace. The final realization is then obtained by a up-projection of the lower dimensional optimal model.



### References

- [1] Sigmund, O. "A 99 line topology optimization code written in MATLAB." Structural and Multidisciplinary Optimization, vol. 21, no. 2, 2001, pp. 120–27.
- [2] S. Hu, C. Yuan, A. Castagnotto, B. Lohmann, S. Bouhedma, D. Hohlfeld, T. Bechtold. "Stable reduced order modelling of piezoelectric energy harvesting modules using implicit Schur Complement". Microelectronics Reliability, 2018, 85, pp. 148-155.

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