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The free jet as a means to improve water quality: Destratification and oxygen enrichment

Jürgen Michele*, **Volker Michele**

Institut für Energie-, Verfahrens- und Umwelttechnik (EVU) der Fachhochschule Oldenburg, Ostfriesland, Wilhelmshaven, Standort Wilhelmshaven, Germany

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Abstract

Rivers, lakes, and coastal waters are chaotic systems – physical, chemical, and biological parameters influence their development. Each parameter itself is influenced by the system. Human interaction has led to fast eutrophication. Oxygen input and artificial mixing have been considered as tools to overcome the biggest problems of fish kills, algal blooms, and bad odour. The favoured technology for destratification and oxygen input so far is the bubble curtain. This technology has been applied successfully in several cases. But often, this technology could not be implemented because of high investment and operating costs.
Alternatively, the free jet is discussed as an efficient and low investment and operating cost technology. The free jet may transport oxygen-rich water from the surface down into the hypolimnion, thereby destratifying a water system. A free jet entrains on its way down even more oxygen rich and warm epilimnic water. This water will finally – if some mixing with the cold hypolimnic water occurs – be transferred to the metalimnion. The density differences will make this water travel long distances.
The energy input may be very low and the objective must not be to totally overturn a system. A jet started in early spring may help a lake to have a deep enough epilimnion, relatively large in volume in respect to the hypolimnion, and the normal wind will recirculate the water transferring enough oxygen to the deeper part, thus expanding the fish habitat and enabling benthic fauna. Literature also shows that the occurrence of massive algal blooms may be reduced.
The oxygen efficiency can be multifold compared to standard technologies.

Key words: Free jet – artificial mixing – destratification – oxygen enrichment – algal bloom – blue-green algae

***Corresponding author:** Prof. Dr.-Ing. Jürgen Michele, Institut für Energie-, Verfahrens- und Umwelttechnik (EVU) der Fachhochschule Oldenburg, Ostfriesland, Wilhelmshaven, Standort Wilhelmshaven, Friedrich-Paffrath-Straße 101, D-26389 Wilhelmshaven, Germany; e-mail: michele@fbm.fh-wilhelmshaven.de

Artificial mixing and oxygen supply technology

Stratification in early spring reduces the circulation of lakes and coastal waters, thereby leading to oxygen depletion in the hypolimnion. Even one warm day may start this situation and, if no strong winds stir up the system, this situation will remain until autumn overturn.

The spring bloom of algae will be finished when the nutrients are consumed. This primary production of algae will sink to the bottom and will be decomposed by bacteria. When the oxygen content in the hypolimnion is lowered, the destruction is accomplished by anoxic species. This situation will be especially bad in eutrophic and hypertrophic water systems, leading to bad odour and fish kills from ammonia, hydrogen sulphide and other toxic substances. Even if these extremes are not observed, the anoxic situation of the lower part of a water system may lead to a complete loss of all benthic fauna. If the bottom surface layer is anoxic, fish eggs will not develop.

To reduce the strain on such an ecosystem, oxygen input and artificial mixing are technologies which may overcome critical situations and help to mineralise the biomass bottom layers.

Nature cannot overcome stratification if the density differences of water are large, see below. In water without salt, the density differences which occur because of the temperature differences, are in the order of 2 kg/m^3, already indicating a strong stratification. Water density is not a linear function of temperature. At higher temperatures, the same temperature difference means a higher density difference. If mixing with sea water occurs, the density differences may be in the order of 24 kg/m^3. When local mixing occurs, these density differences create hydrostatic pressure differences, which in turn will be converted to relatively high velocities, which will spread the mixed water over long distances.

Strong winds, even of long duration, will affect the thickness of the epilimnion, but usually do not reach down to the ground. The velocities typically generated are in the order of 10 cm/s. This velocity is much too low to overcome a stronger stratification, even if converted to the vertical direction.

A number of technologies have been developed to overcome these problems. The reduction of nutrients is the long term objective, but will not be discussed in this article. The systems in use may be divided in two groups: input of energy by mechanical pumping and air or pure oxygen lift systems. Depending on the goal, the technologies aim at total mixing or only for oxygenation of the hypolimnion. The last method is relatively expensive, but is usually favoured when a reservoir is used as a drinking water supply in order to keep the water cool.

If a water system with no density gradient is mixed completely, then the slightest wind will mix it totally and keep it in a mixed state. Mechanical work has to be done to avoid new stratification tendencies. Less energy is needed to keep a system in the mixed state compared to the energy needed to cure a strong stratification.

Most pumping systems conduct fluid from the lower hypolimnion to the surface or vice versa in long tubes. These only lead to increased friction and are not necessary. To reduce the power input, systems are of large diameters, but this increases installation costs dramatically.

Air lift systems, bubble curtains, are considered to be effective and efficient and are proposed by administration offices. The authors of this paper are convinced, as will be shown, that a free jet will also do a good job if properly designed, and at considerably lower investment and operating costs.

The free jet

If one fluid is introduced into another at a different speed, the resulting stream may be termed a free jet. Generation may be realised by mechanical pumping, by a waterfall, or by bubbles creating a bubble plume.

Because of its technical importance, this subject is well studied. Applications are general mixing of liquids and gases, combustion, sprays, etc.

General well known properties

Theoretical and experimental results may be used in connection with similarity laws to calculate the effect of a free jet leaving a circular orifice. Results date far back.

Equations for calculation of jet properties can be found in works by PERRY (1997), SCHLICHTING (1965), TRUCKENBRODT (1992), ZILCH et. al. (2002), OERTEL (2001) and others. There are only slight variations in the experimentally determined coefficients.

The following equations are adapted from TRUCKENBRODT (1992). These represent a round turbulent jet of diameter, d_0, leaving an orifice. The initial volume flow rate, Q_0, is increased considerably by entrainment on its way downstream. The momentum remains constant, meaning that the center line velocity of the bell shaped distribution curve is reduced. These equations apply to the fully developed region about six initial diameters downstream of the orifice. They hold only for the same properties of the jet itself and the surrounding medium. The jet is determined by its momentum. The propagation of a jet is considerably different when the viscosities and the densities of the liquids are different.

The jet radius, r, at a downstream distance, x, from the orifice can be calculated from equation [1]. By definition, at r the velocity is half the maximum value at the center line.

$$r = 0.085 \cdot x \qquad [1].$$

These equations hold true only for $x > 6\,d_0$. Initial behaviour is dominated by the starting conditions.

The maximum jet velocity is given by:

$$u_{\max} = 6.57 \cdot \frac{d_0}{x} \cdot u_0 \qquad [2],$$

where u_0 is the exit velocity of the jet at the orifice. The reach of the jet depends on its initial momentum.
The volume flow rate may be computed from the following equation.

$$Q_x = 0.456 \cdot \frac{x}{d_0} \cdot Q_0 \qquad [3].$$

The velocity distribution at a lateral distance, x, and radial distance, r, may be calculated from the center line speed with the following equation.

$$u = u_{\max} \cdot \exp\left(-\ln 2 \cdot \left(\frac{r}{d}\right)^2\right) \qquad [4].$$

In order to appreciate the order of magnitude, one has to insert numbers.

From an energy standpoint, one would make use of large diameter jets with low exit velocities. This would lead to low operating but high installation cost. A low velocity will not be sufficient to overcome a stronger stratification, as will be shown below.

A realistic installation may start with the following conditions: $d_0 = 1$ m, $u_0 = 1$ m/s.

In this case, computation will deliver the following results:

Fig. 1 shows the radius development of the jet – it starts with the initial value and increases slightly to the diameter calculated downstream. Here the diameter is defined by the location where the speed is decreased to half the maximum at the center line.

The decrease in center line velocity is shown for this special case in Fig. 2. Up to 7 meters downstream the velocity at the center remains at 1 m/s and will only be reduced to 0.219 m/s already 30 m downstream.

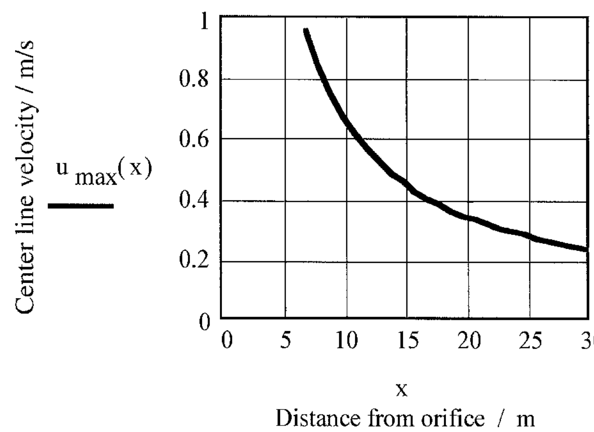


Fig. 2. Center line velocity.

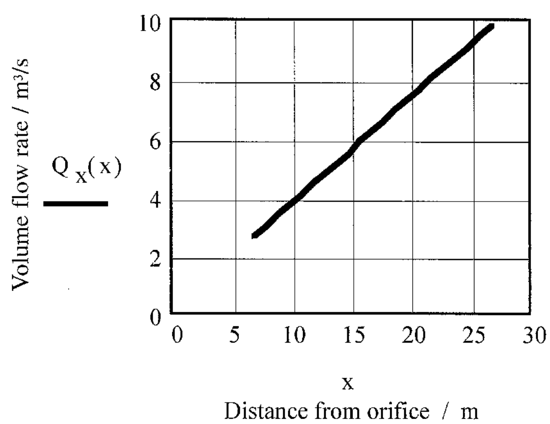


Fig. 3. Volume flow rate.

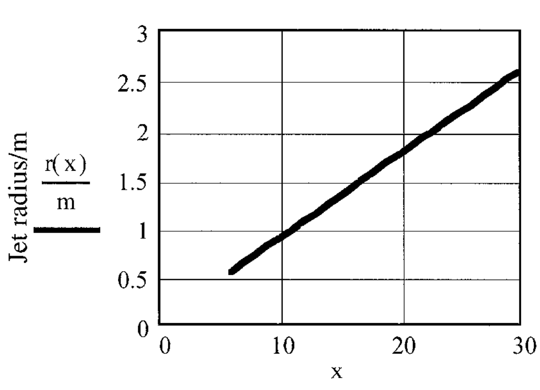


Fig. 1. Jet downstream diameter (for definition see text).

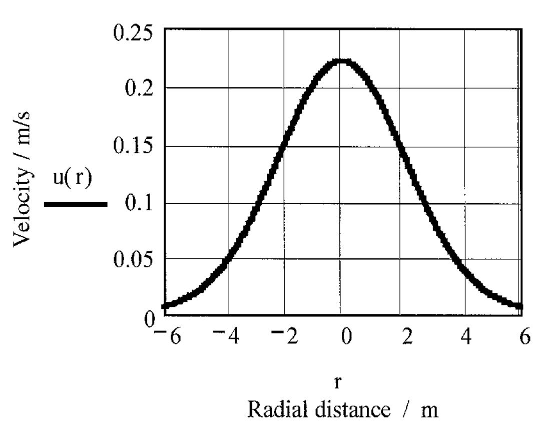


Fig. 4. Velocity distribution after 30 m of travel.

The volume flow rate at the starting point is Q_0 = 0.785 m³/s. Fig. 3 shows that the volume flow rate is increased after a distance of 30 m to Q = 10.7 m³/s. This means that the moving volume has increased more than ten times.

Furthermore the velocity after a distance of 30 m is still relatively high. Fig. 4 shows the velocity distribution.

Power requirements to generate a straight forward flow are very low. To generate the discussed jet, only 392 Watts are theoretically required. Another way to appreciate this low power is that it is equivalent to the power needed to pump the initial flow rate Q_0 = 0.785 m³/s to a height of only 5.1 cm.

If a jet is set free in an area with high oxygen levels, it is capable of bringing down large volumes of water to deeper oxygen-depleted locations. The oxygen supply efficiency can be very high.

Note again that these calculations are valid only for a not-stratified fluid system.

Special features of circular jets

A special but also well studied feature of a jet is the Coanda effect [see e.g. TRITTON (1988)]. A jet will be attracted to a nearby surface and will stick to it further downstream if the curvature is not too sharp. This also means that two jets will attract each other. As explanation the Bernoulli effect may be offered: In a turbulent case, if no entrainment is possible or reduced because of the geometry on one side, the jet will move to this side. A horizontal surface jet will be drawn down after a relatively short distance. Experimental observations, measurements, and numerical results are found for example in the work of NASR & LAI (2000).

Not so well known is the fact that a round jet will induce a spiralling motion. This effect, a secondary flow, is reported by LEHMANN (1986). He obtained this result from laser Doppler measurements on a free air stream. This means that in addition to the forward and outward motion, there is a circular motion of the jet. There is only one vortex dominating the structure of flow which travels, spirals and grows. Micro-mixing happens in smaller scale vortices.

This behaviour is of special importance for the mixing properties of circular jets, which will be shown in the experimental part of this article.

Experimental model studies

Because similarity laws can be used efficiently to study jet behaviour, experiments were conducted in a small aquarium, 80 cm × 40 cm × 40 cm. The objective of the experimental work was to visualise the principal behaviour, mixing properties, influence of the angle of the jet direction, and the effect of stratification.

The spiralling motion can be seen in Fig. 5. Here, a laminar water jet marked with ink is released from a pipe. Because of deceleration, the motion becomes turbulent after a short distance.

This behaviour can also be observed in Fig. 6a and b, where a jet leaves a 16 mm inner diameter pipe at a speed of 0.2 m/s – this corresponds to a Reynold's number of 3300.

In Fig. 6a the jet is totally inked, whereas in Fig. 6b ink is added at the top of the jet. After only one jet diameter some ink shows up at the bottom of the jet, indicating a partial overturn and demonstrating the above discussed spiralling motion of round jets.

The Coanda effect can be seen in Fig. 7. The parameters are the same as in Fig. 6, but a steeper angle was set. The jet sticks to the plate and becomes flatter.

If a too strong jet is released close to the bottom it may stir up the fine bottom sediments – especially sunken algae. This may create oxygen and visibility problems in the water. On the other hand, this is only a local effect which will be overcome in a very short time if the jet is not too strong and creates erosion problems with coarser and heavier sediments.

The main reason for installation of a jet is the destratification of a water system. Therefore, experiments were conducted with our laboratory system incorporating density differences.

If a jet of low density at a high temperature is released into a denser stagnant fluid, this jet will eventually reverse its direction and rise again.

For an order of magnitude estimation, this effect can be calculated using Bernoulli's law – see basic fluid mechanic literature, for example OERTEL (2001). The only assumptions required in this special case are: there is no friction, and the density remains constant on the center line. These assumptions hold quite well in practical cases.

In this special case, this leads to an equation which gives the maximum travel distance of the center line fluid:

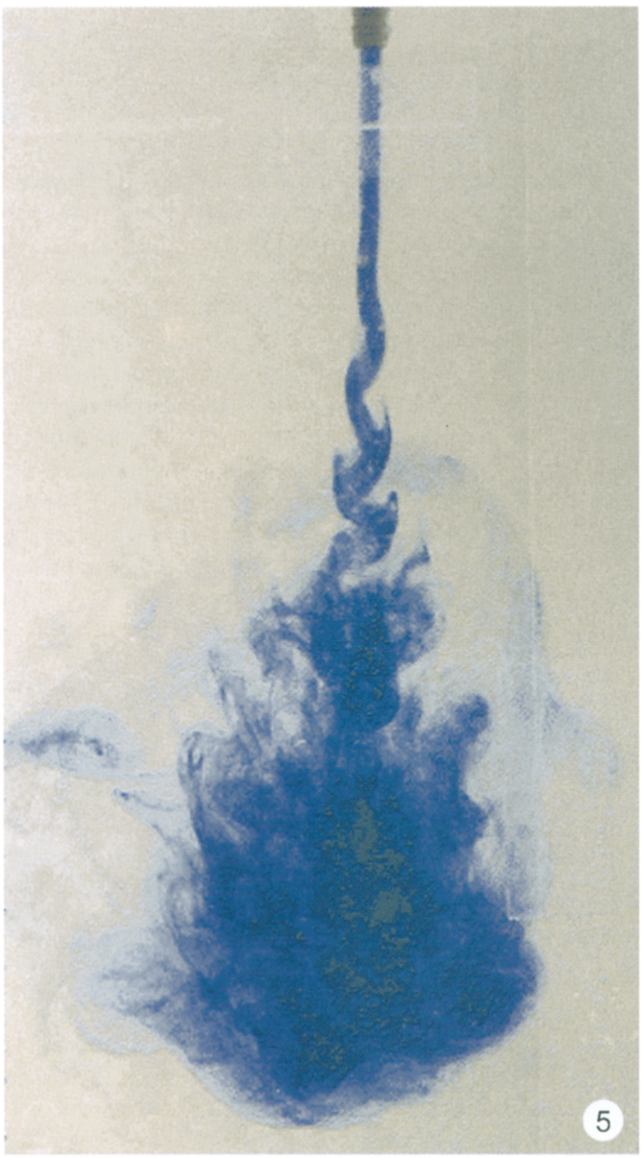
$$\Delta h = \frac{1}{2} \cdot \frac{\rho}{\Delta\rho} \cdot \frac{u^2}{g} \qquad [5]$$

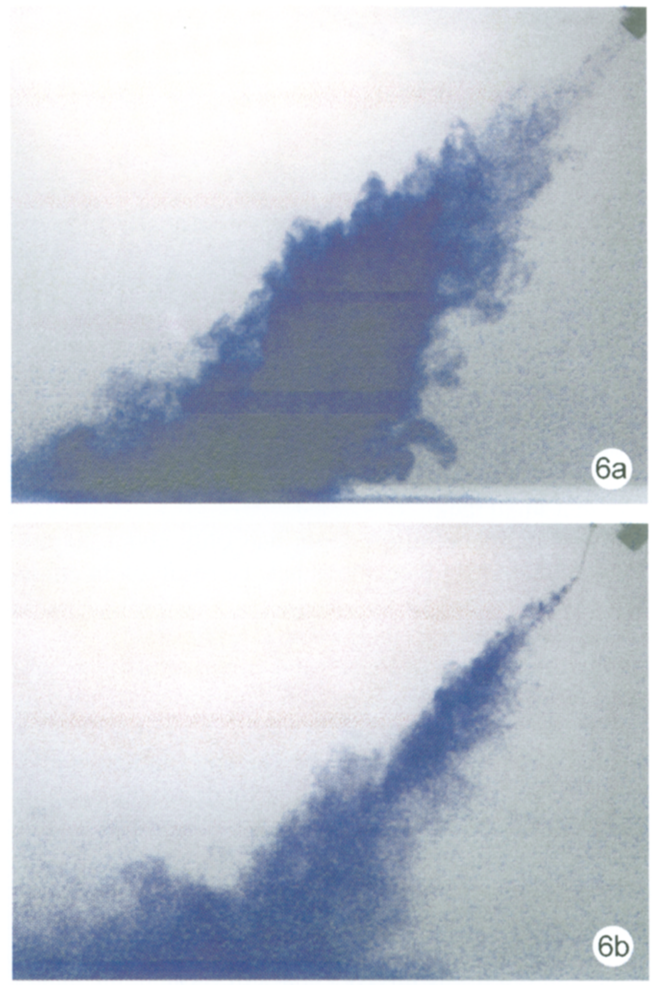
Here are: Δh = length of the pass in the denser medium,
ρ = density of the fluid,
$\Delta\rho$ = density difference,
u = velocity at the point of release into the denser fluid,
g = gravitational constant.

To estimate the order of magnitude, this equation has been computed for a typical water stratification with a density difference of 2 kg/m³. The result is given in Fig. 8.

This straightforward equation shows the importance of initial velocities for a destratification operation. In order to avoid the wasting of energy, the starting velocity was only 5 cm/s in some cases reported in the literature. For this condition, the above equation will give a further penetration of only 6.4 cm. This means the jet will not reach further down from the release point. On the other hand, a speed of 1 m/s will penetrate additional 25 m. Because of the simplifications a jet will not reach this deep – entrainment reduces the distance.

Fig. 9 shows a vertical buoyant jet. In order to have turbulent flow and also to be able to demonstrate the effect in an aquarium, the operating conditions were as follows:





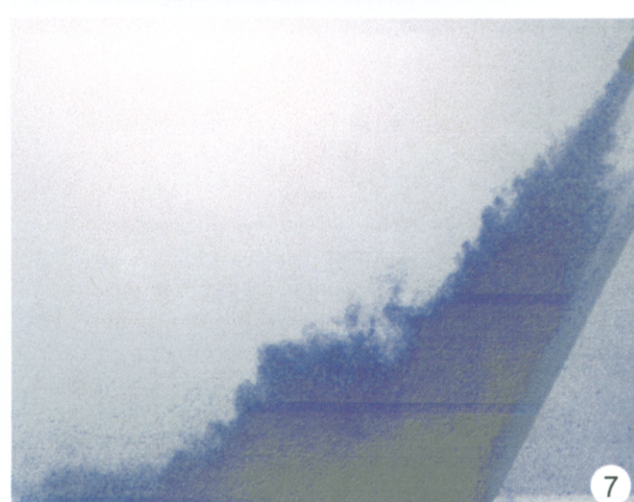


Fig. 5. Spiraling motion of a laminar buoyant water jet marked with ink. Unpublished results by DOMINGO PAGEY (2002).

Fig. 6. Jet at 45° angle – Reynold's number: Re = 3300. **a.** Ink was added to the primary flow. **b.** Ink was added into the upper region of the free jet.

Fig. 7. Coanda effect – the jet is attracted to the plate (see text for details).

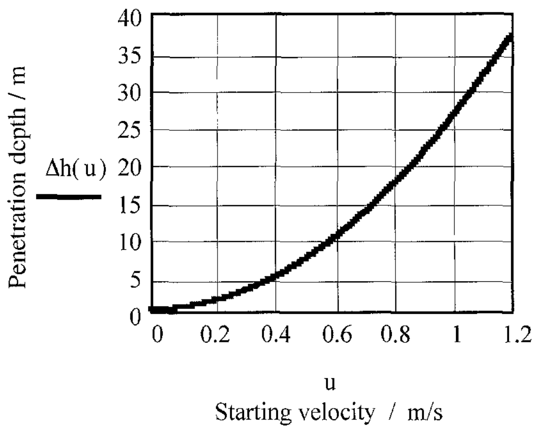


Fig. 8. Penetration depth for a density difference of 2 kg/m³.

density difference = 6.0 kg/m³,
orifice diameter = 16 mm,
exit velocity = 0.20 m/s.
This resulted in a Reynold's number of 5000 and a volume flow rate of 150 l/h.

These values were necessary to demonstrate this effect in a small tank.

The final penetration depth was only about 20 cm. The equation [5] given above has the result of 34 cm in this case.

Fig. 10 shows a jet inclined at an angle of 45°.

The comparison of Figs. 9 and 10 proves that the vertical arrangement has to be avoided if it is intended to accomplish destratification. In a very short time the vertical jet mixes only with itself. The entrainment comes from the rising fluid. Only a very small mixing process is realised at the outer surface of the slowly rising plume.

The advantages of an inclined jet are evident. Up to the lowest point, intensive mixing occurs with the low-temperature, high-density fluid. This mixing will mean that a large amount of the mixed water will settle in the metalimnion and will not rise all the way back to the surface.

Which discharge direction to choose – design hints

It has been shown above that a vertical jet is not the proper way to solve destratification problems. The inclination should be such that the oxygen rich water of lower density mixes with the hypolimnic water. This will help to widen the metalimnion. The normal wind situation will help to keep the metalimnion well mixed and oxygen rich. Nature will add most energy to the mixing process. The momentum, M, of the jet is calculated by the following equation and is responsible for the reach of the transferred fluid,

$$M = \dot{m} \cdot u_0 = p \cdot A_0 \cdot u_0^2 \quad [6]$$

where: $\dot{m}$ = mass flow, A_0 = Area – pipe cross section.

Surface winds are most important for the movement of the liquid by inducing a surface stream. Back flow has to occur; an inclined jet should be installed in such a manner as to increase this back flow. It is best located at the prevalent down wind side of a lake and directed to increase this recirculation. This also means that oxygen rich water is driven by wind and wave action to the suction side of a properly designed destratification system. Such a system is proposed in a patent application – German Patent Application (2001).

Numerical simulation by LÜCKING (2002, person. commun.) confirms the efficiency of such a system. The design of the system itself is important if maximum efficiency is desired. Fig. 11 shows the flow around a possible installation geometry. For simplification, only a vertical system was designed to make use of symmetry.

It is evident that the flow to the destratification unit is mainly from the nearby surface. If the suction layer is not too thick, oxygen will be dissolved by mass transfer from the atmosphere in low oxygen situations of the upper water layer. In addition, large masses of water streaming by the unit, are being sucked in by the free jet. This means that the momentum of the jet is increased even by the momentum of the entrained flow. This is not taken into account in the equations given above.

As may be derived from an energy balance, it can also be concluded, that it makes no sense to intermittently operate such a system. It takes a relatively long time to bring the system into recirculation and achieve the required strong mixing effects. Wind driven currents will strongly interact.

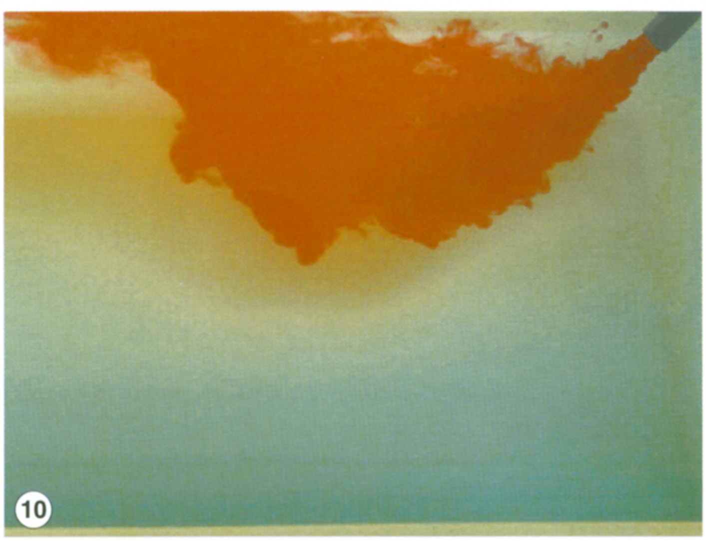
In the design of such a destratification system one has the choice of diameter and speed to generate the desired momentum. A larger diameter will require less energy for the same momentum and the same reach. Thus one could decide to opt for large installations. This, however, would mean a high installation cost.

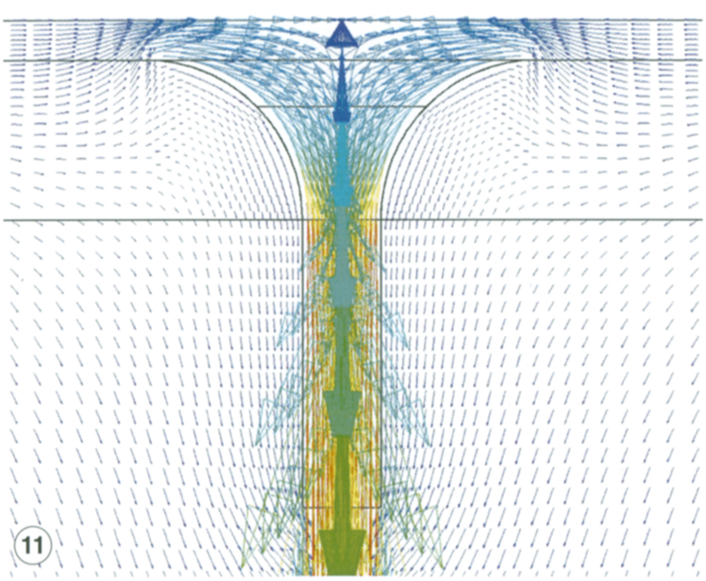
The example given above seems to be of reasonable size. Because the mixed fluid has to travel, it may be useful to install other units in distant locations, especially if there are deeper, separated parts in a lake system.

The strength of the jet can also be adjusted by inclination and controlled propeller speed. This can avoid stirring up sediments. However, the introduction of oxygen into the sediments may lead, in the long run, to mineralisation.

Comparison to other technologies

The restoration and managing of eutrophic lake systems are long term topics of limnological literature (WETZEL





2001; JAEGER & KOSCHEL 1995; HORNY & GOLDMAN 1994; KLAPPER 1992; COOKE 1986, and many others).

Mechanical mixing and oxygen introduction are considered possible means to overcome especially severe problems. On the other hand, human interaction should be minimised. The costs of such technical systems are usually high due to the installation, operation, and maintenance effort of the equipment.

Generally, the technology of bubble curtains is considered very efficient. This has been demonstrated in a report edited by Thüringer Landesanstalt für Umwelt in Jena (2000). The operation of this successful system was nevertheless discontinued because of high operational and maintenance costs.

There is no doubt that the efficiency of an air lift system in respect to the mere pumping of water is considerably low. The compression of air itself has an already low efficiency value compared to pumping of incompressible liquid. The main disadvantages are:

1. At the point of release of the air the volume flow rate of water is practically zero. An equation for calculation of the water flow rate is given by PASTOROK et al. (1982). This means that the water at this location is not mixed efficiently. There will be practically no mixing below the injection point.
2. A bubble plume also causes a free jet.
   The difference from a "normal" jet is that its momentum keeps increasing until it reaches the surface. Here, the strongest mechanical action is observed at a location where it is not needed.
3. The usually assumed oxygen input at the water surface does not happen.
   The observed effects are generally from mixing with the upper level fluid which usually has a high oxygen level, and the mass transfer from the injected oxygen.

Fig. 9. Vertical buoyant jet.

Fig. 10. Inclined buoyant jet.

Fig. 11. Surroundings of a vertical destratification unit (stationary conditions) – flow vectors (LÜCKING 2002, person. commun., unpubl. results).

The rising fluid may leave the jet if its density in the outer part of the jet is changed by mixing with warmer water and its momentum there is not strong enough to overcome the density stratification, see e.g. ASAEDA & IMBERGER (1993) and also above.

The horizontal installation of an air distribution system is of special importance for the uniform supply of air. That usually means that installation at the deepest part of a lake system is not possible.

The technology of hypolimnic oxygenation is also connected with strong mixing. The suction, and in particular the discharge of the larger volumes of water, which are needed to be efficient causes this mixing. If the jets are released symmetrically from the systems, as found in the literature, this causes mixing from top to bottom, caused by the normal horizontal direction of the jets themselves – see above. Also, the entrainment will induce a strong vertical downstream of warm water to the hypolimnion. It is no wonder that LINDENSCHMIDT (1998) reports this effect in his PhD thesis, where he found the modelling of a lake system leading to better model results when he modelled "the hypolimnic aerator as a hypolimnic stirrer".

The efficiency can be discussed in terms of oxygen efficiency. The value for "standard" systems used in waste water treatment are in the order of up to 4 kg O_2/kWh. ZLOKARNIK (1999) reports in his book on mixing several technologies for the gasification of liquids. In particular, he looks at the efficiency of technical systems.

The efficiency of bubble plumes is in the same order of magnitude.

The main difference between a lake and waste water treatment is that the hold up times in a lake are much longer and the oxygen requirements are not so high. Thus, the oxygen transport into the hypolimnion may be accomplished by a relatively small stream of oxygen rich water. If taken directly from the surface, this ensures that it is normally not only saturated but even over-saturated by the oxygen production of phytoplankton.

At this point the question is raised, what the maximum efficiency of oxygen transport could be. It may be shown that when the technology just mentioned is applied, this value may be infinite if the downward speed is zero for an infinite mass of water being transported to an area which is oxygen depleted.

Above it has been shown that, because of stratification, the flow will eventually reverse. Thus, a close to "zero" speed is not possible and also very large installations would be required.

The application of the proposed technology may have an efficiency, which is more than one order of magnitude better compared to the established technologies. The power needed to create a large momentum jet is low. No costly installations for piping and supports are needed. This means considerable economical advantages, when the treatment of lakes and ponds is necessary. For even larger sea areas, this technology may be applied. Bays of the Baltic Sea suffer from oxygen depletion and the proposed technology could help the situation at reasonable costs.

A main advantage of a destratified system is, that even low wind creates a recirculation which transfers oxygen to the bottom layer. The proposed jet technology can deliver oxygen all the way down to the bottom layer. Benthic fauna is made possible again and the habitat for life forms needing oxygen is expanded.

A disadvantage of destratification should be mentioned: the mixing will lead to an increase of temperature. This may mean that the solubility of oxygen is reduced at the higher temperature. On the other hand, if there is no oxygen in an eutrophic lake, all benthic fauna will disappear and oxygen supply is the only way to overcome the urgent problems.

Effects on algal blooms

The effect of mixing on algal blooms, especially cyanobacteria, is discussed in the limnological literature. HYENSTRAND et al.(1998) reviewed recent work (some 162 references). Artificial mixing is also discussed by VISSER et al. (1996 a and b), there the successful operation of a bubble plume is discussed.

There seems to be general agreement that mixing and oxygen input are not capable of solving all cyanobacteria problems. There are reports that, upon starting a mixing operation, massive fish kills occurred. Nevertheless, the authors of the present paper are convinced that, mixing technology, if applied properly, will lead to improved water quality in the long run.

Conclusions

The free jet is a very efficient tool to convey large amounts of oxygen rich water into the metalimnion, thereby expanding the epilimnion and destratifying a water system as intended. The installation and operation costs are very low, so this technology can be applied for coastal basins, large and smaller lakes and even fish farming ponds. The oxygen efficiency may be more than one order of magnitude higher compared to other technologies. If properly designed for the depth required the jet reaches down – even in strong stratification situations (saline layers). This means that the habitat for oxygen demanding life forms is expanded. Generally, the literature reports increased biodiversity if mixing is achieved and the habitat is oxygen rich.

Installation should be such that the natural effects are reinforced and not reduced.

With an inclined free jet the mass transport is threefold: primary flow by pumping, entrainment, and mixing of cold (dense) water by the raising stream. The moved mass is increased considerably over the initial stream.

Starting the operation in early spring would have the greatest effect. The first spring destratification would be destroyed and the development of it will start later in the year. Artificial mixing generally leads to an early overturn in autumn. When the anoxic hypolimnion is kept relatively small, no negative effects will be observed in this situation.

Required energy levels may be so low that consideration of photo voltaic or wind-driven operation of the pumping system is worthwhile.

Acknowledgements

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