Script EC Rezekne January 2019 J. Timmerberg

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Electric Circuits

(10) 1.) DC Network
(16) 2.) AC network, 1 phase
(10) 4.) AC network, 3 phase

48 h
32 h...
36 h lessons

~~5.) Induction motors~~

• Physical Values

= quantity · unit

↓ we can measure

↘ SI-System

in SI-System: kg, m, s, A, K, rad...

Basic Units →

• Multiples + Submultiples of units

T : Tera 10^{12}
G : 10^{9}
M : 10^{6}
k : kilo 10^{3}
m : 10^{-3}
μ : micro 10^{-6}
n : 10^{-9}
p : pico 10^{-12}

• Unit operator

mass : m

$[m] = kg$; $[t] = s$

$[a] = ?$ $a = \frac{dv}{dt}$; $[a] = \left[\frac{dv}{dt}\right] = \frac{\Delta v}{\Delta t} =$

$[v] = ?$ $v = \frac{ds}{dt}$; $[v] = \left[\frac{ds}{dt}\right] = \left[\frac{\Delta s}{\Delta t}\right] = \frac{m}{s}$

$[a] = \left[\frac{dv}{dt}\right] = \left[\frac{v}{t}\right] = \frac{m}{s \cdot s} = \frac{m}{s^2}$

~~I = 4 amp~~

$I = 4\ mA$

$I = \frac{P}{t} = \cancel{\frac{4W}{3s}} = \frac{4}{3}\cancel{\frac{W}{s}}$

Units

Basic elements in electrical eng.

material consists of electrons, protons, neutrons $\frac{m_P}{m_E} \approx 10.000$

Charge electron → $q_e = -1.6 \cdot 10^{-19}\ As$
$= -1.6\ 10^{-19}\ C$
$\Rightarrow 1\ C = 1\ As$

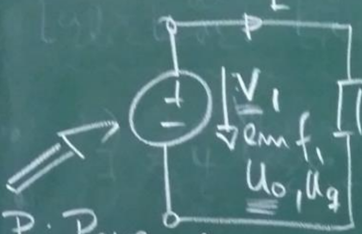
Charge proton → $q_p = +1.6 \cdot 10^{-19}\ C$
↳ elementary charge

$\boxed{Q = n \cdot q_e}$ charge in electrical eng.
↳ Integer

Charge is the basic of electrical eng.

→ current definition: $\boxed{I = \frac{dQ}{dt}}$ Spec: $I = \frac{Q}{t}$; $[Q = I t] = As = C$

• Sources: voltage source, current s.



P: Power
W: Work } energy, work ≠ electrical

P, W: thermal, mechanical, optical, ...

normally: P = 5W (±) → P (±)

→ W (±) → $\frac{W}{Q}$ (±) → $\boxed{V}$ (±)

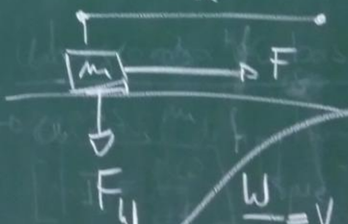
• Power

[P] = W

[W] =

$\left[\frac{W}{Q}\right] = \frac{J}{C} = \frac{Ws}{As} = \frac{VA}{A} = V$

$\left[W = F \cdot s = F \cdot d\right] = \boxed{Nm = J = Ws}$



$\frac{W}{Q} = V$

V: Potential Difference

Very important

$\boxed{1\ Ws = 1\ J = 1\ Nm}$

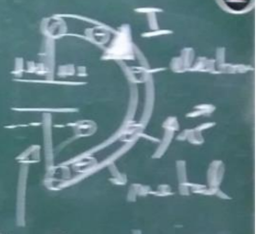
Example 2.2: 300 mC

V = 100 V

$W = Q \cdot V = 300\,mC \cdot 100V$

$= \frac{300 \cdot As \cdot 100V}{1000}$

$= 30\ VAs = 30\ Ws$



P, W

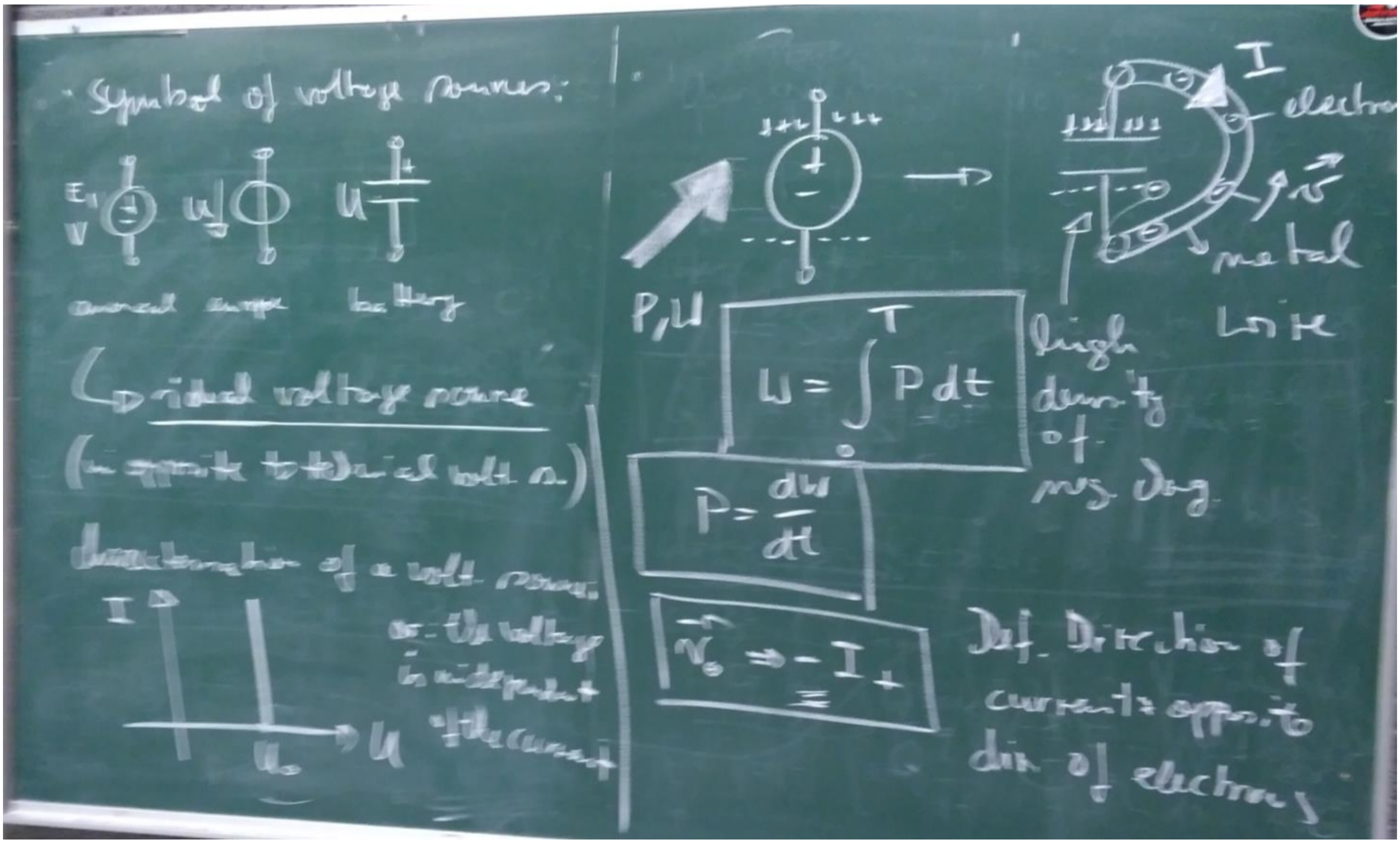
$\boxed{W = \int_0^T P\,dt}$

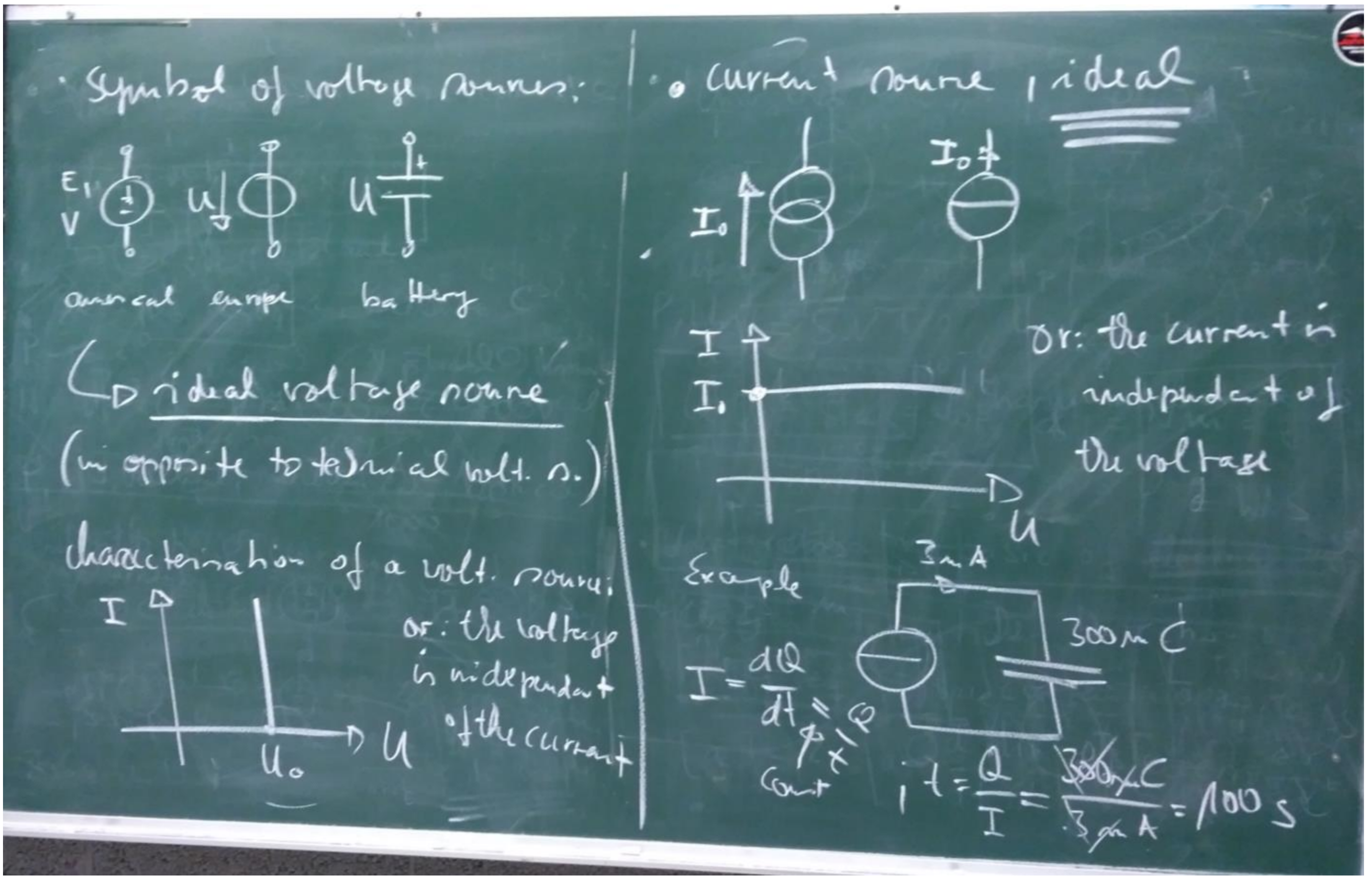
length density of mov. charge

$\boxed{P = \frac{dW}{dt}}$

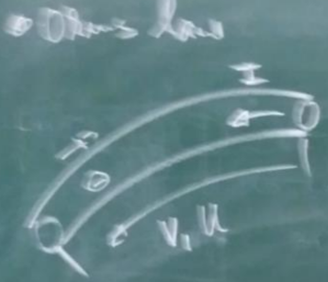
$\boxed{\vec{v}_{e} \Rightarrow -I_{+}}$

Def. Direction of current is opposite to dir. of electrons


· Symbol of voltage sources:
E, V
U
U
battery
↳ ideal voltage source
characterisation of a volt. source:
I
U₀
U
or: the voltage is independent of the current
P, U
$W = \int_0^T P\,dt$
$P = \frac{dW}{dt}$
$\vec{v}_0 \Rightarrow -I_+$
I
electr
metal
wire
high density of
Def. Direction of current ≠ opposite to dir. of electrons


· Symbol of voltage sources:
E, V
U
U
american
europe
battery
↳ ideal voltage source
(in opposite to technical volt. s.)
characterisation of a volt. source:
I
U₀
U
or: the voltage is independent of the current
· current source, ideal
I_0
I_0
I
I_0
U
or: the current is independent of the voltage
Example
3mA
300mC
$I = \frac{dQ}{dt}$
const
$t = \frac{Q}{I} = \frac{300\,mC}{3\,mA} = 100\,s$

• Ohms law

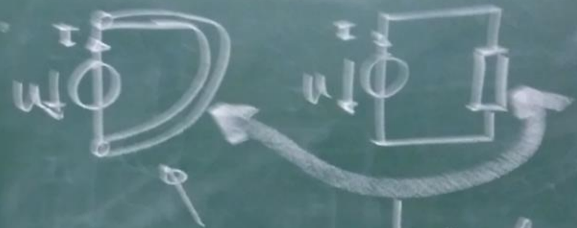


Ohm: $V = U \sim I \Rightarrow$ prop.

$\left\{ \begin{array}{l} F \sim a \Rightarrow \\ \text{Eq. } F = ma \end{array} \right.$

$\boxed{V = U = RI}$

⇒ in experiment

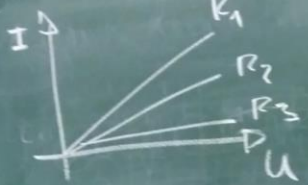


complicated | simple

the box R stands for the wire, a general wire. We call it resistor R

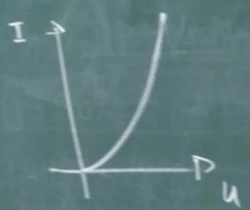
Ohm: $\boxed{R = \frac{U}{I} = \text{const}}$

$[R] = \left[\frac{U}{I}\right] = \frac{V}{A} = \Omega$; example $R = 7\,\Omega$



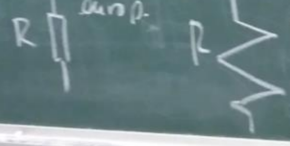
all Rs are linear

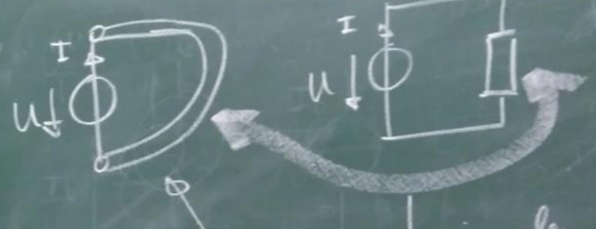
$R_1 < R_2 < R_3$



R is not linear

Symbol of R





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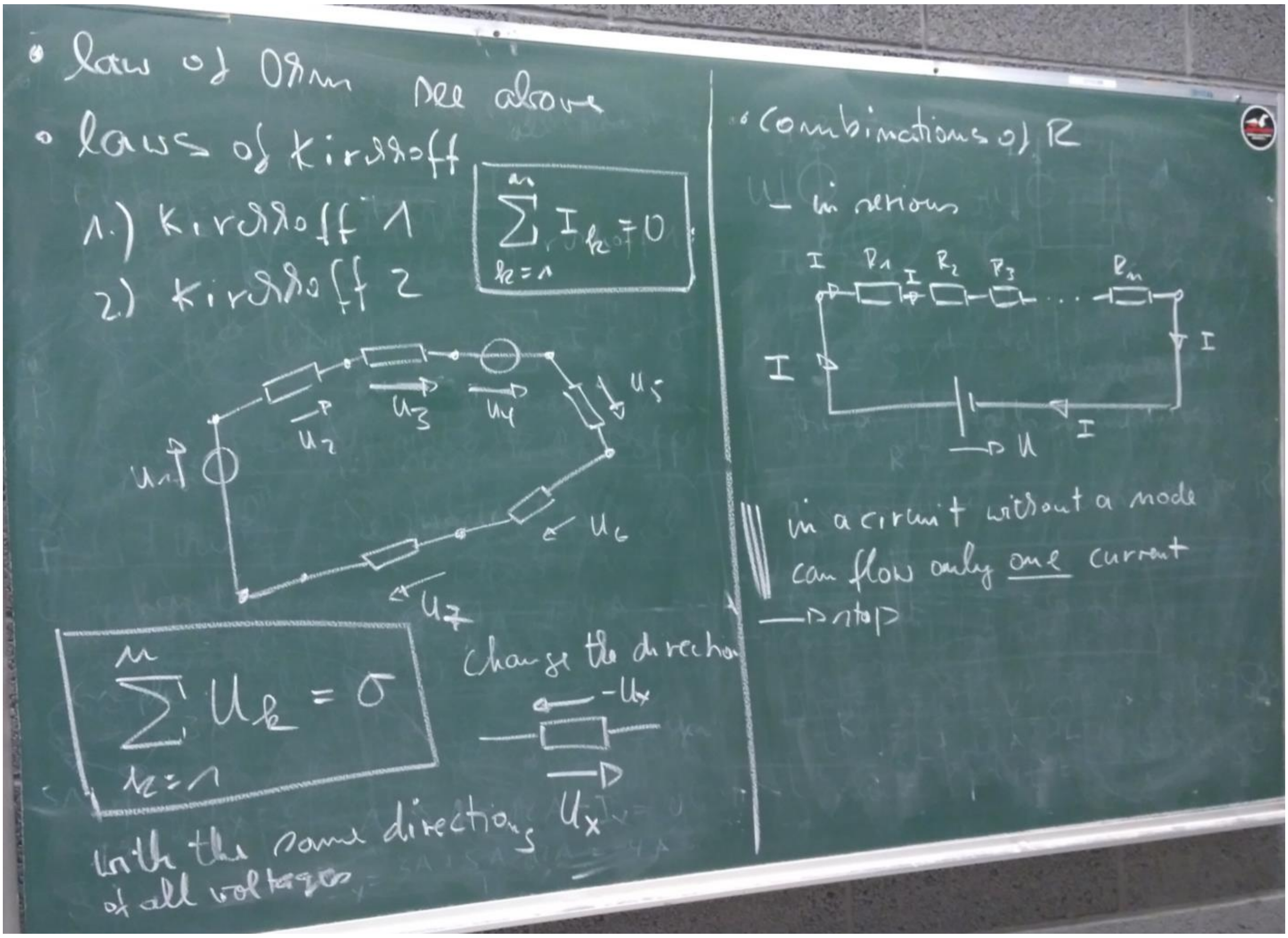
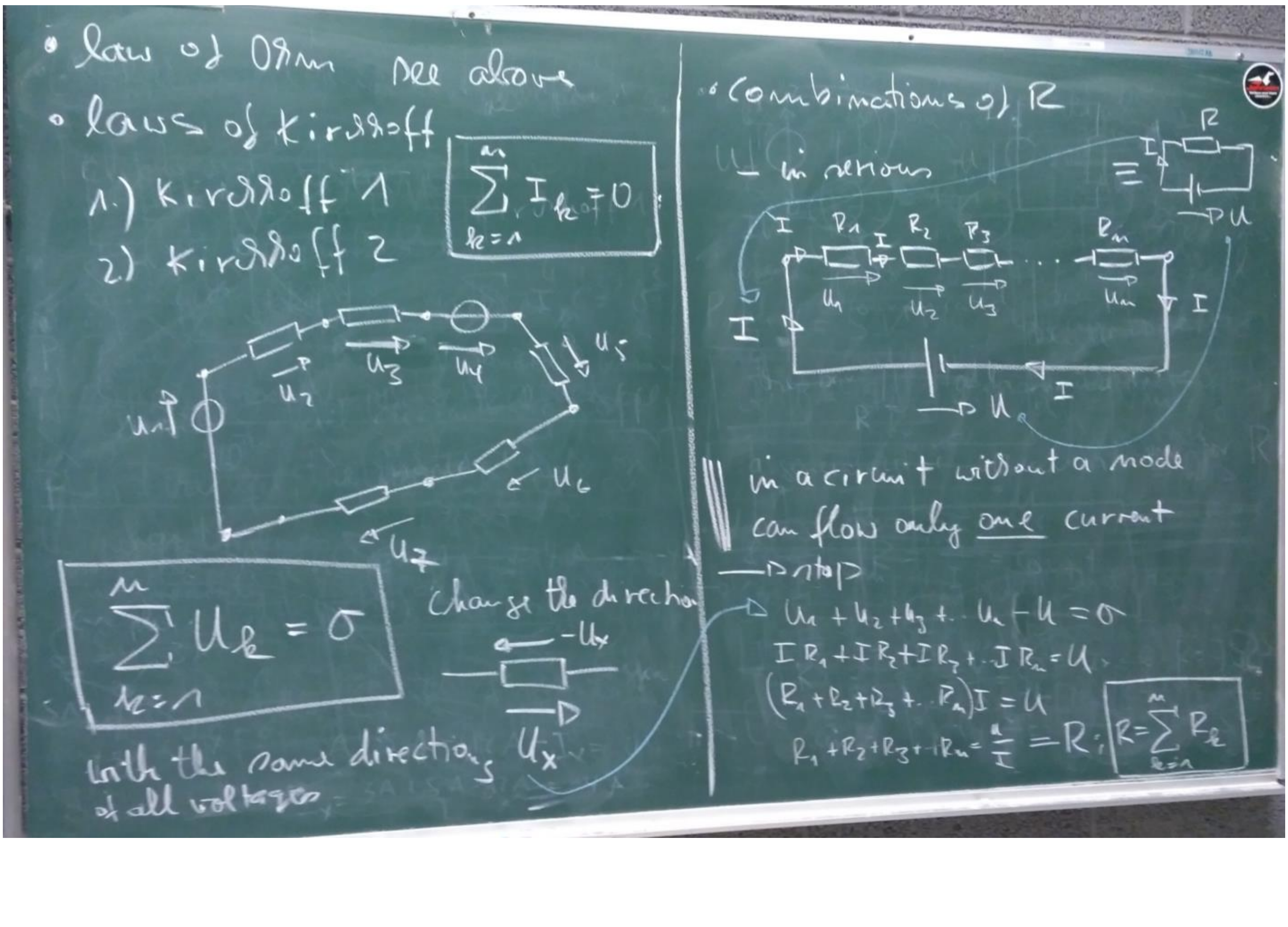

law of Ohm see above
laws of Kirchhoff
1.) Kirchhoff 1
2.) Kirchhoff 2
Σ I_k = 0 (k=1 … n)
u1 u2 u3 u4 u5 u6 u7
Σ U_k = 0 (k=1 … n)
change the direction
-Ux
Ux
with the same directions of all voltages
combinations of R
– in serious
I R1 I R2 R3 Rn
I
I
I
U
in a circuit without a node can flow only one current
→ stop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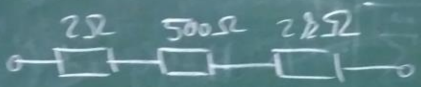
combinations of R
– in serious
R
I
≡
U
I R1 I R2 R3 Rn
U1 U2 U3 Un
I
I
I
U
in a circuit without a node can flow only one current
→ stop
→ U1 + U2 + U3 + .. Un − U = 0
I R1 + I R2 + I R3 + .. I Rn = U
(R1 + R2 + R3 + .. Rn) I = U
R1 + R2 + R3 + .. Rn = U/I = R ; R = Σ Rk (k=1 … n)

example:



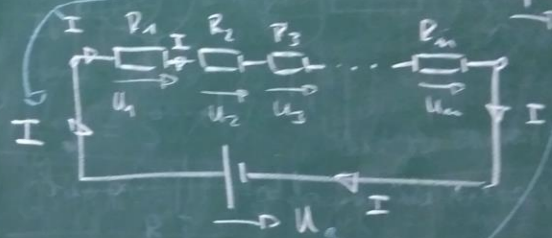
What is the subresistor for

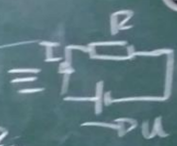
$$R = 2\Omega + 500\Omega + 2k\Omega$$

$$= \underline{\underline{2502\,\Omega}}$$

• Combinations of R

— in series





in a circuit without a node can flow only one current

→ step

$$U_1 + U_2 + U_3 + \dots U_n - U = 0$$

$$IR_1 + IR_2 + IR_3 + \dots IR_n = U$$

$$(R_1 + R_2 + R_3 + \dots R_n) I = U$$

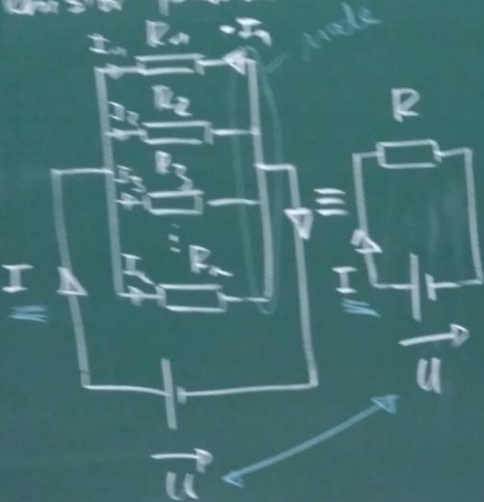
$$R_1 + R_2 + R_3 + \dots R_n = \frac{U}{I} = R; \quad \boxed{R = \sum_{k=1}^{n} R_k}$$

• Ohms Law

• Kirchhoff 1, 2

• Resistor in series

• Resistor parallel



$$\sum_{k=1}^{n} I_k = I_1 + I_2 + I_3 + \dots I_n - I = 0$$

equal to: $-I_1 - I_2 - I_3 \dots - I_n + I = 0$

$$\frac{\cancel{U}}{R_1} + \frac{\cancel{U}}{R_2} + \frac{\cancel{U}}{R_3} + \dots \frac{\cancel{U}}{R_n} - \frac{\cancel{U}}{R} = 0$$

$$V = R \cdot I, \quad I = \frac{V}{R}$$

$$\boxed{\frac{1}{R} = \sum_{k=1}^{n} \frac{1}{R_k}}$$

$$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \frac{1}{R_n} = \frac{1}{R}$$

spec. case: $n = 2$:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \; ; \; R = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \cdot \frac{R_1 R_2}{R_1 R_2}$$

$$R = \frac{R_1 \cdot R_2}{\frac{R_1 R_2}{\cancel{R_1}} + \frac{R_1 R_2}{\cancel{R_2}}} = \boxed{\frac{R_1 \cdot R_2}{R_1 + R_2} = R}$$

spec.

example:

$$\frac{1}{R} = \frac{1}{7\Omega} + \frac{1}{3\Omega} = \ldots \text{dif.}$$

$$R = \frac{7\Omega \cdot 3\cancel{\Omega}}{10\cancel{\Omega}} = 2.1\,\Omega$$

$$\sum_{k=1}^{n} I_k = I_1 + I_2 + I_3 + \ldots I_n - I = 0$$

equal to: $-I_1 - I_2 - I_3 \ldots - I_2 + I = 0$

$$\frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3} + \ldots \frac{U}{R_n} - \frac{U}{R} = 0$$

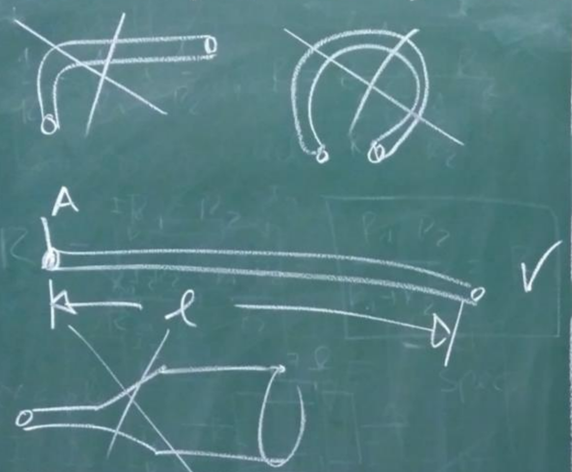
$$V = R \cdot I \; , \; I = \frac{V}{R}$$

$$\boxed{\frac{1}{R} = \sum_{k=1}^{n} \frac{1}{R_k}}$$

general

$$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \frac{1}{R_n} = \frac{1}{R}$$

• Resistivity of a straight wire



$$\boxed{R = \rho \cdot \frac{l}{A} = \frac{1}{\sigma} \cdot \frac{l}{A}}$$

σ = conductivity in $\frac{m}{\Omega}$

ρ = specific resistance in $\frac{\Omega}{m}$

$$\boxed{\sigma = \frac{1}{\rho}} \quad \boxed{G = \frac{1}{R}} \text{ — conductivity}$$

$$[R] = \Omega = \frac{V}{A}$$

$$[G] = \frac{1}{\Omega} = \frac{A}{V} = S$$

!

example: A wire with the length of 32m and the area $A = 3\,cm^2$ of Cu: how high is R?

$$R = \frac{m \cdot 32km}{6 \cdot 10^7 S \cdot 3\,cm^2} =$$

$$\sigma_{Cu} = 6 \cdot 10^7 \frac{S}{m} \quad \text{page 16}$$

$$= \frac{\Omega\, \cancel{m} \cdot \cancel{3000m}}{\cancel{6} \cdot \cancel{10^7} \cdot 3 \cdot \cancel{10^{-4}}\, m^2}$$

$$= \frac{1}{6}\,\Omega$$

σ = conductivity in $\frac{m}{\Omega}$

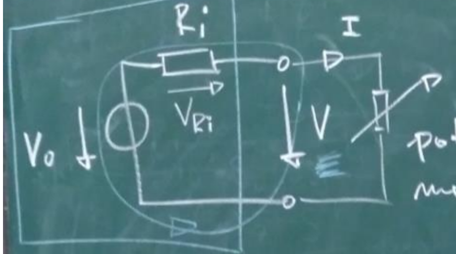
ρ = specific resistance in $\frac{\Omega}{m}$

$$\boxed{\sigma = \frac{1}{\rho}} \quad \boxed{G = \frac{1}{R}} \text{ conductivity}$$

$$[R] = \Omega = \frac{V}{A}$$

$$[G] = \frac{1}{\Omega} = \frac{A}{V} = S \quad !$$

- Real sources
  - real voltage source



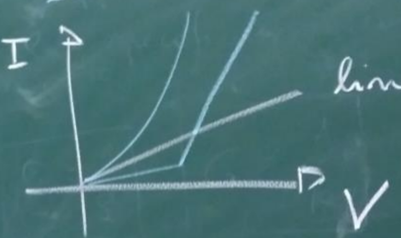
potentiometer

OR

may be PV-cell

R_i = internal resistor

non linear resistor



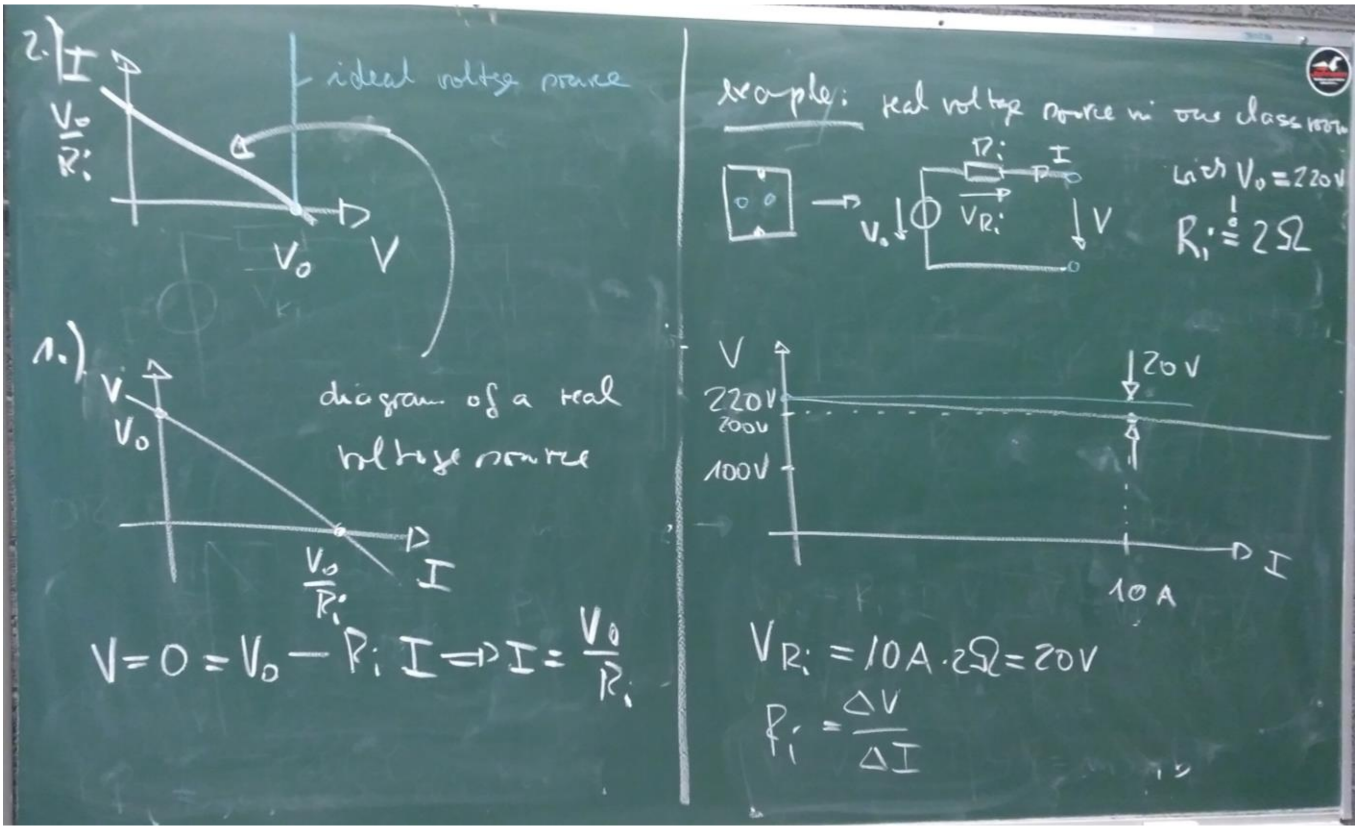
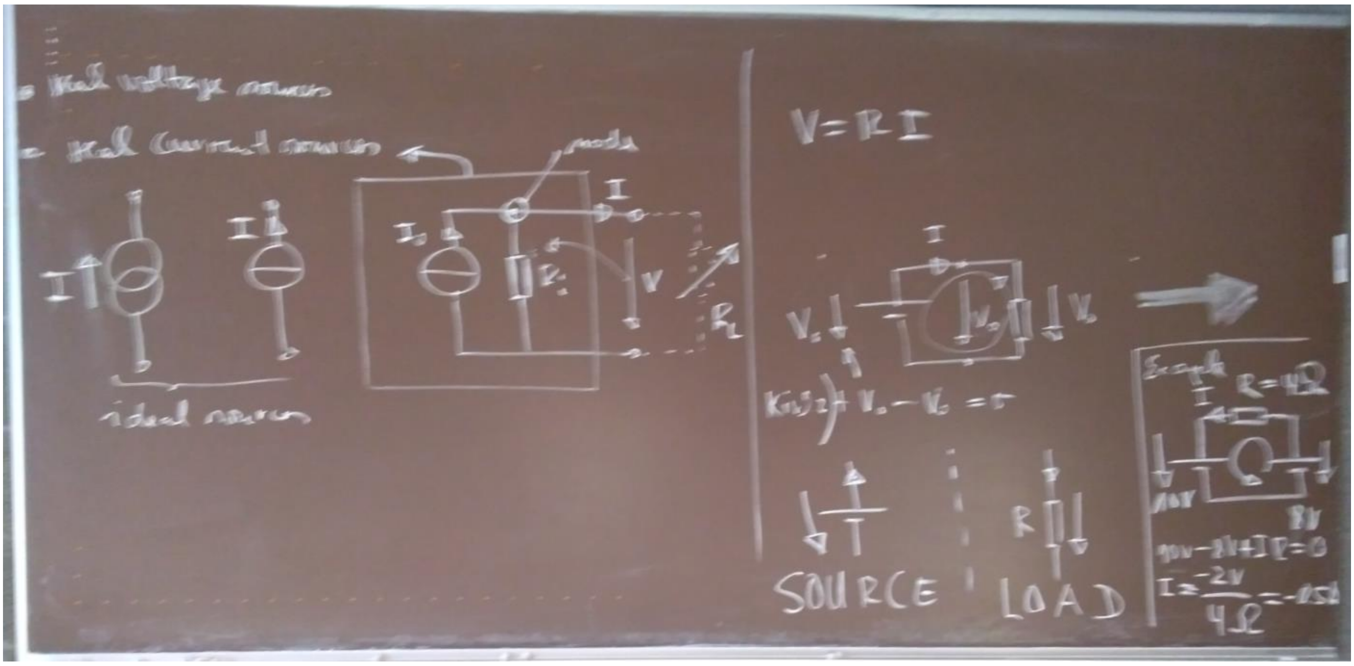
we take R_i as linear (first)

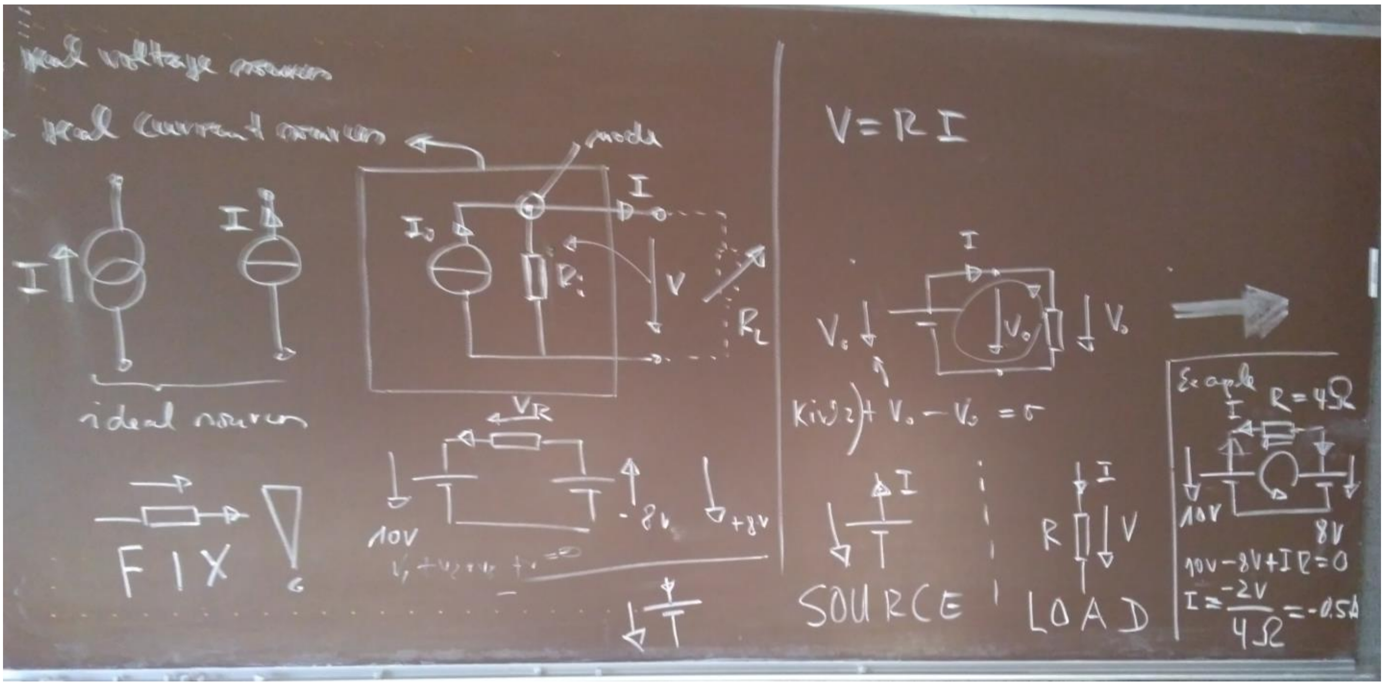
→ we want to calculate $V(R_i, V_0)$

$$V_{R_i} = R_i \cdot I \;;\; \sum_k V_k = V_{R_i} + V - V_0 = 0$$

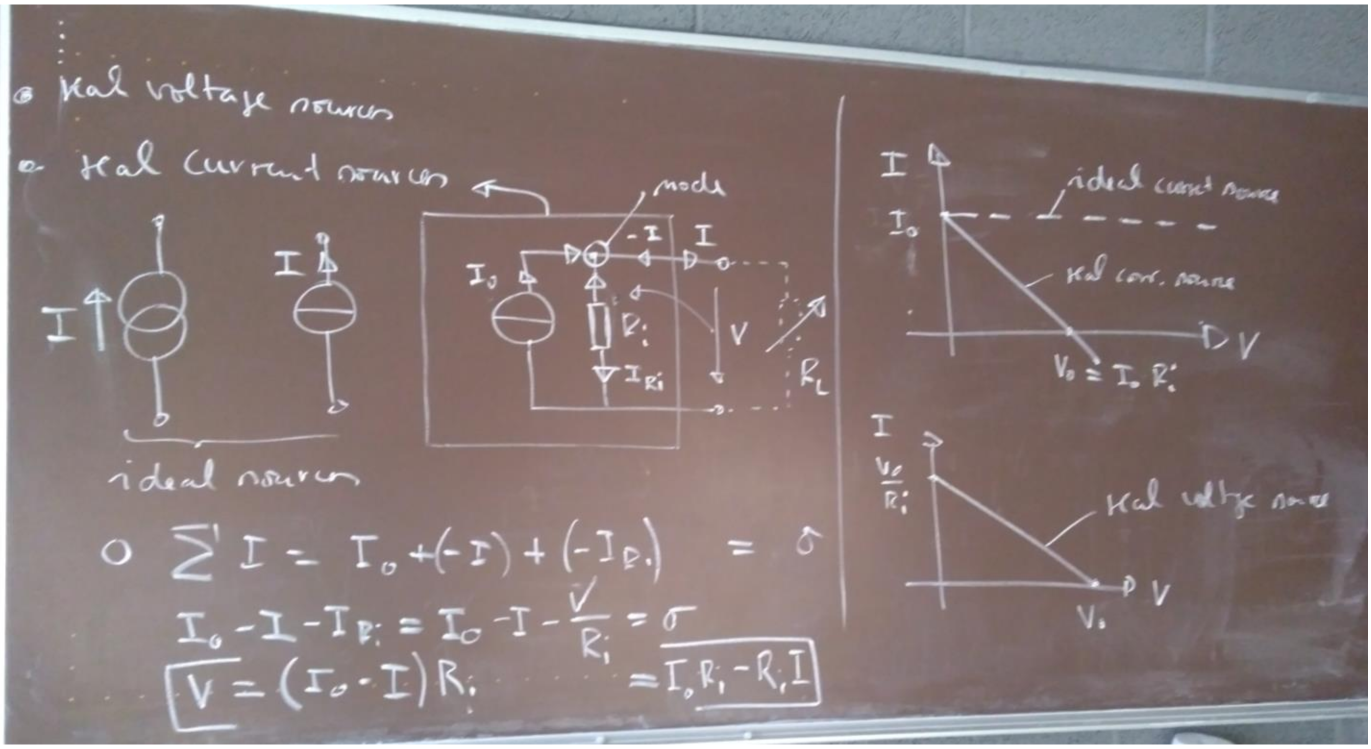
$$V = V_0 - V_{R_i} = V_0 - R_i I$$

$$\boxed{V = V_0 - R_i I}$$


2.)
ideal voltage source
1.)
diagram of a real voltage source
V = 0 = V0 − Ri I => I = V0 / Ri
Example: real voltage source in our classroom
with V0 = 220 V
Ri = 2 Ω
220 V
200 V
100 V
20 V
10 A
VRi = 10 A · 2 Ω = 20 V
Ri = ΔV / ΔI

real voltage source
real current source
node
ideal source
V = R I
SOURCE
LOAD
Example
R = 4 Ω
I = −2V / 4Ω = −0.5 A



real voltage sources
real current sources
node
I
I_0
R_i
V
R_L
ideal sources
V_R
10V
-8V
+8V
FIX
V = R I
V_0
$+V_0 - V_0 = 0$
SOURCE
LOAD
Example
R = 4Ω
8V
10V-8V+IR=0
I = -2V / 4Ω = -0.5A

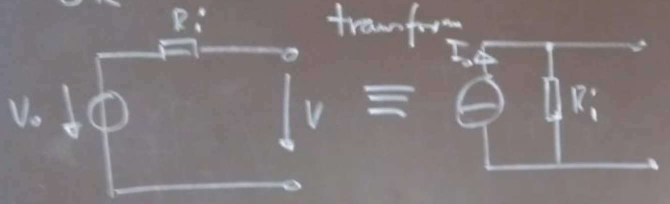


real voltage sources
real current sources
node
-I
I
I_0
R_i
V
I_{R_i}
R_L
ideal sources
$\sum I = I_0 + (-I) + (-I_{R_i}) = 0$
$I_0 - I - I_{R_i} = I_0 - I - \frac{V}{R_i} = 0$
$\boxed{V = (I_0 - I) R_i} = \boxed{I_0 R_i - R_i I}$
ideal current source
I_0
real curr. source
$V_0 = I_0 R_i$
$\frac{V_0}{R_i}$
real voltage source
V_0

- real voltage sources
- real current sources

you can transform a real voltage source into a real current source, if $\boxed{V_0 = I_0 \cdot R_i}$

OR



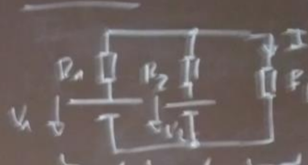
Example:

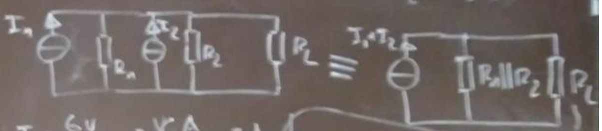
$R_1 = R_2 = 2\Omega$

$R_L = 4\Omega$

$V_1 = 6V$

$V_2 = 5V$



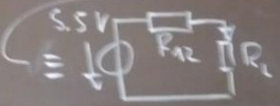


$I_1 = \frac{6V}{2\Omega} = 3\frac{V \cdot A}{V} = 3A$

$I_2 = 2.5 A$

$I_1 + I_2 = I_3 = 5.5 A$

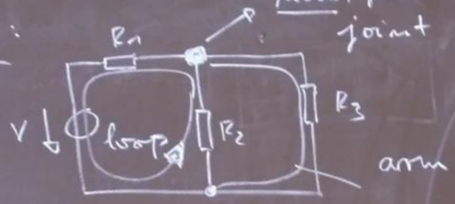
$R_1 || R_2 = R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} = 1\Omega$



$I = \frac{5.5V}{5A} = 1.1 A$

- real current source
- abstract circuit analysis

definition:

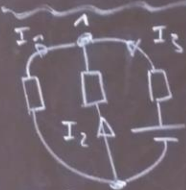


loop: closed, has a (free) direction

arm: consists of one resistor and a voltage source and ends in two nodes

= closed path over arms and nodes.

independent equations:



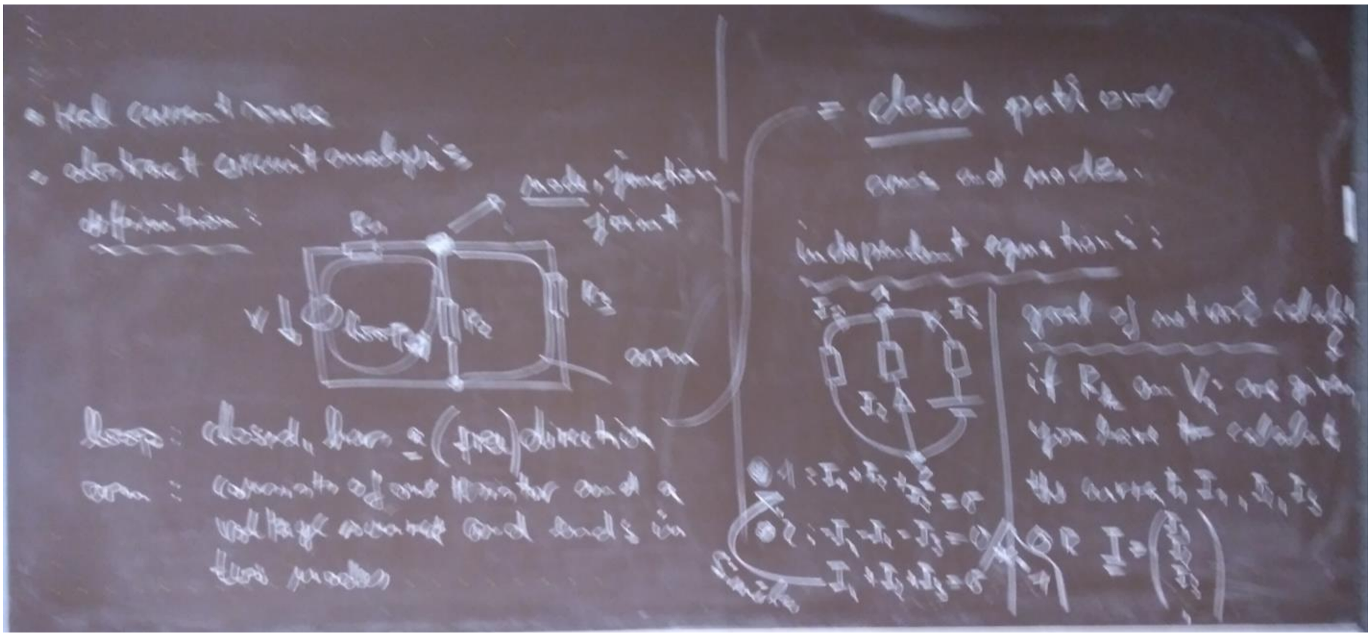
1: $I_1 + I_2 + I_3 = 0$

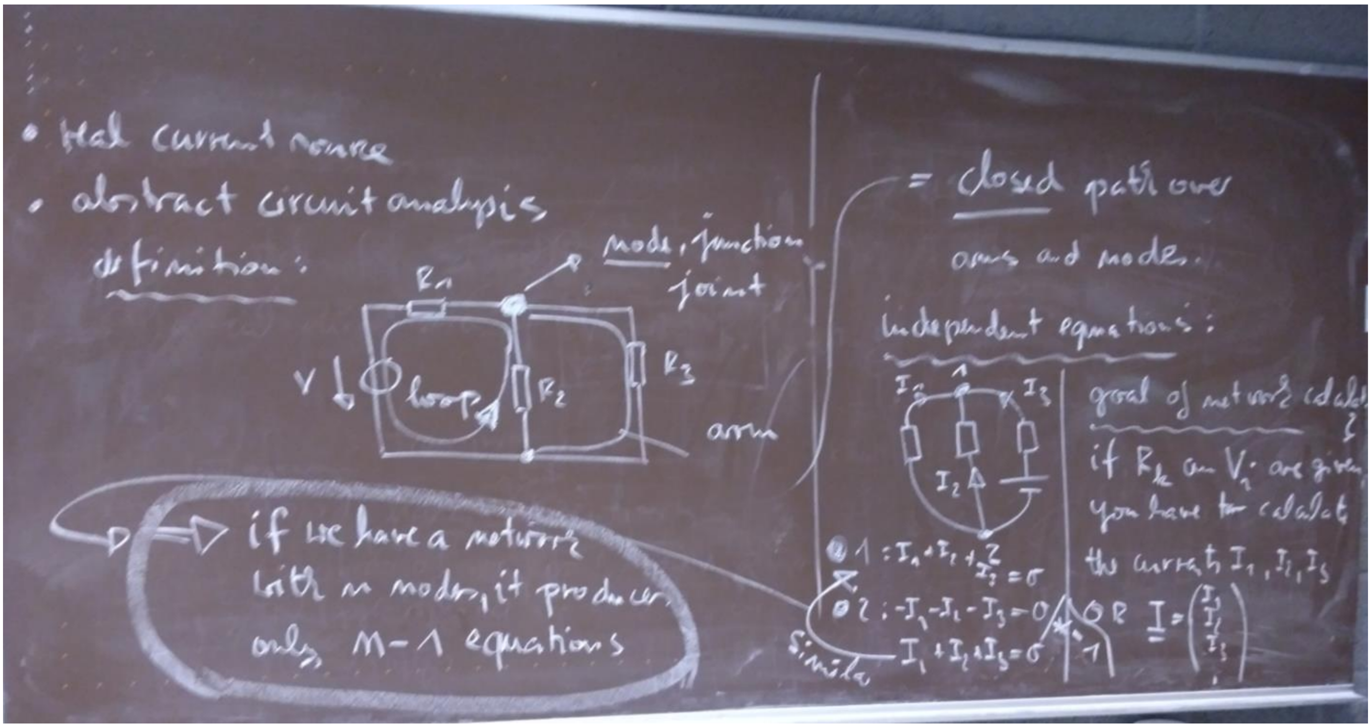
2: $-I_1 - I_2 - I_3 = 0$

similar $I_1 + I_2 + I_3 = 0$

goal of network calculation?

if R_k and V_i are given, you have to calculate the currents I_1, I_2, I_3 OR $I = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$


• Hall current source
• abstract circuit analysis
definition:
node, junction, joint
R1
R2
R3
V
loop
arm
loop: closed, has a (free) direction
arm: consists of one resistor and a voltage source and leads in two nodes
= closed path over arms and nodes.
independent equations:
goal of network calculation
if R_k and V_i are given you have to calculate the currents I1, I2, I3
similar


• Hall current source
• abstract circuit analysis
definition:
node, junction, joint
R1
R2
R3
V
loop
arm
if we have a network with n nodes, it produces only n−1 equations
= closed path over arms and nodes.
independent equations:
I1
I2
I3
1 : I1 + I2 + I3 = 0
2 : −I1 − I2 − I3 = 0
similar
I1 + I2 + I3 = 0
goal of network calculation
if R_k and V_i are given you have to calculate the currents I1, I2, I3
I = (I1 I2 I3)

Very important

$$\boxed{1\,Ws = 1\,J = 1\,Nm}$$

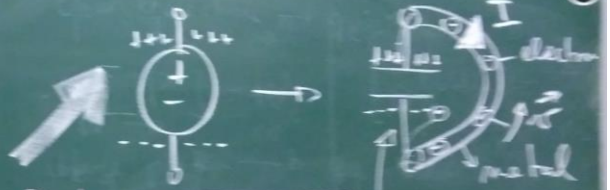
Example 2.2 : 300 mC

$V = 100\,V$

$W = Q \cdot V = 300\,mC \cdot 100V$

$= \frac{300 \cdot As \cdot 100V}{1000}$

$= 30\,VAs = 30\,Ws$



P, W

$$\boxed{W = \int^T P\,dt}$$

$$\boxed{P = \frac{dW}{dt}}$$

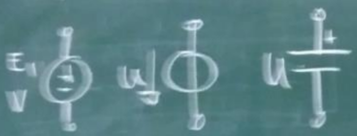
$$\boxed{\vec{v}_e \Rightarrow -I_+}$$

high density of mov. charg.

wire

Def. Direction of current is opposite to dir. of electrons

Symbol of voltage sources:



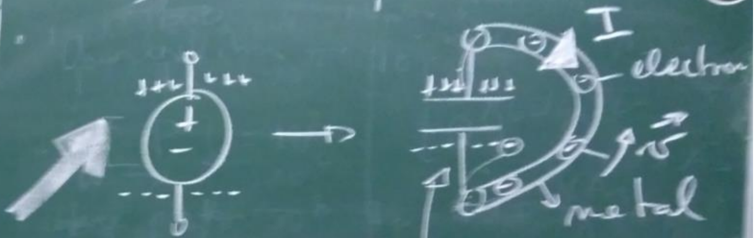
general europe battery

↳ ideal voltage source

(in opposite to technical volt. s.)

characterisation of a volt. source

i.e. the voltage is independent of the current



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high density of mov. charg.

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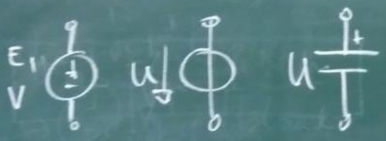
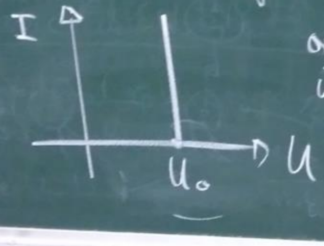


american europe battery

↳ ideal voltage source

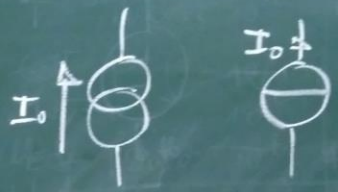
(in opposite to technical volt. s.)

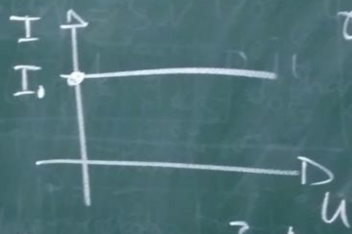
characterisation of a volt. source:



or: the voltage is independent of the current

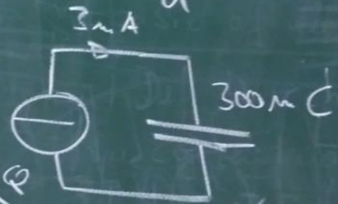
- current source, ideal





or: the current is independent of the voltage

Example



$I = \frac{dQ}{dt} = \frac{Q}{t}$

const

$; \ t = \frac{Q}{I} = \frac{300\,mC}{3\,mA} = 100\,s$