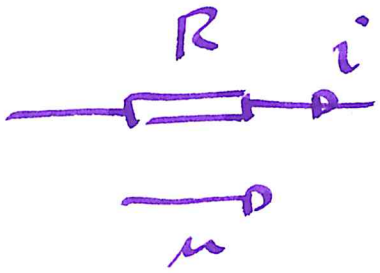
4.4.18, EC-Reference

(DC)

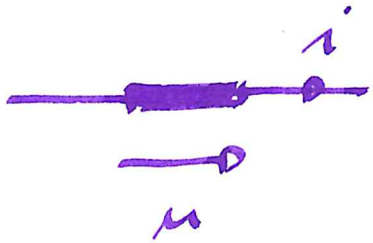
- History



$u = Ri$

$i = \frac{1}{R} u$

$u = L \frac{di}{dt}$



$i = \frac{1}{L} \int u \, dt$

$i = C \frac{du}{dt}$

$u = \frac{1}{C} \int i \, dt$

Alternating current and voltage (AC)

see appendix 1.1

- Complex algebra

~~$i = \sqrt{-1}$~~ $\underline{j} = \sqrt{-1}$

$i, u, p \ldots$ in AC

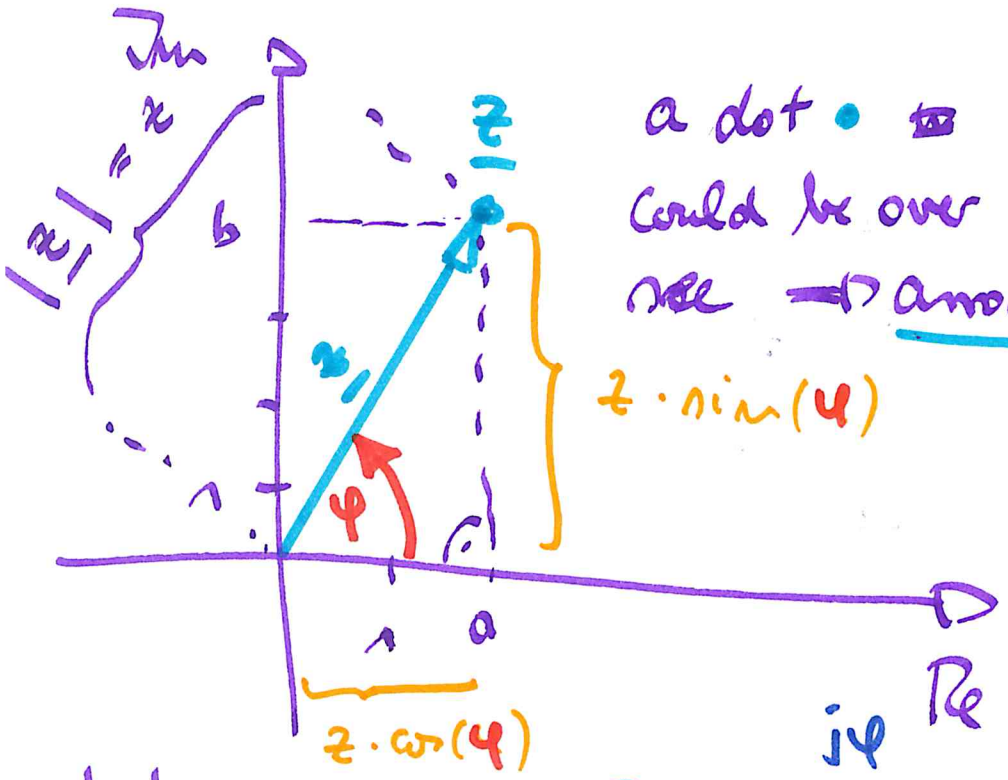
$I, U, P \ldots$ in DC

$\underline{z}$ is a complex no.:

$\underline{z} = a + jb = r \cdot e^{j\varphi}$

cartesian form — Euler form

- Cartesian form vs. Euler form ③



a dot • could be over use ⇒ arrow

$$|\underline{z}| = z \; ; \; \underline{z} = \underline{z} = z \cdot e^{j\varphi}$$

$$\begin{cases} \underline{z} = 2 + j\,4 \\ \underline{e}^{j\varphi} = \cos(\varphi) + j\,\sin(\varphi) \end{cases}$$

④

Taylor: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} +$

$\underline{e}^{jx} = 1 + jx + \frac{(jx)^2}{2!} + \frac{(jx)^3}{3!} +$

$\frac{(jx)^4}{4!} + \frac{(jx)^5}{5!} \cdots$

$= 1 + jx - \frac{x^2}{2!} - j\frac{x^3}{3!} +$

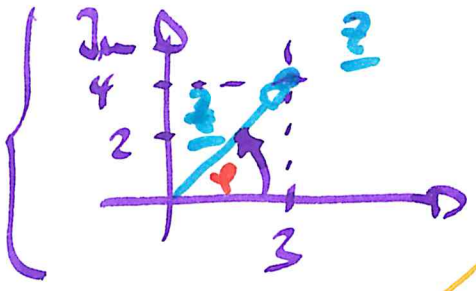
$+ \frac{x^4}{4!} + j\frac{x^5}{5!} \cdots$

$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$ ← cos

$+ j\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots\right)$ ← sin

Example: $\underline{z} = 3 + j4 =$

Transform it into the Euler form.



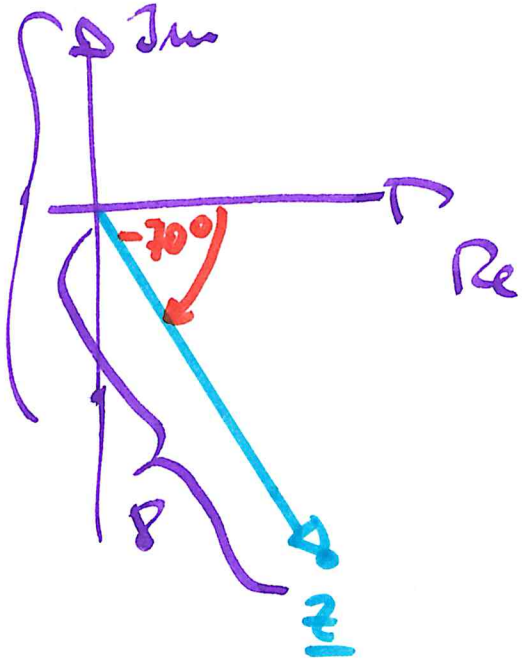
$$z = |\underline{z}| = \sqrt{4^2 + 3^2} = 5$$

$$\angle \underline{z} = \arg(\underline{z}) = \arctan\left(\frac{4}{3}\right) = 53.1°$$

$$= 5 \cdot e^{j 53.1°}$$

↑ Euler form

Example 2: $\underline{z} = 8 \cdot e^{-j 70°}$



Transform it into cartesian form

$$\underline{z} = 8 \cdot e^{-j 70°} = 8 \cdot (\cos(-70°) + j \sin(-70°))$$

$$= 8 \cdot (\cos(70°) - j \sin(70°))$$

$= 8 \cdot (0.342 - j\,0.94)$

$= 2.736 - j\,7.518$ ///

cartesian form

- Production of AC voltage

see appendix 2.1

see page 4:

$\Phi(\alpha) = \hat{\Phi} \cdot \cos(\alpha) =$

- def. velocity: $v = \frac{s}{t} \left(= \frac{ds}{dt}\right)$; $s = v \cdot t$
- $w = \frac{\alpha}{t}$: angle velocity ; $\alpha = \omega \cdot t$

$\hat{\Phi} \cdot \cos(\omega t) = \Phi(\omega t)$

Induction law:

$u = N \frac{d\Phi}{dt} = N \frac{d}{dt} \left(\hat{\Phi} \cdot \cos(\omega t)\right)$

$= N \hat{\Phi} \frac{d \cos(\omega t)}{dt}$

$= -\omega \hat{\Phi} \cdot N \sin(\omega t) =$

u_0

$\left[-\omega \hat{\Phi} \cdot N\right] = \frac{1}{s} Vs \cdot 1 = V$

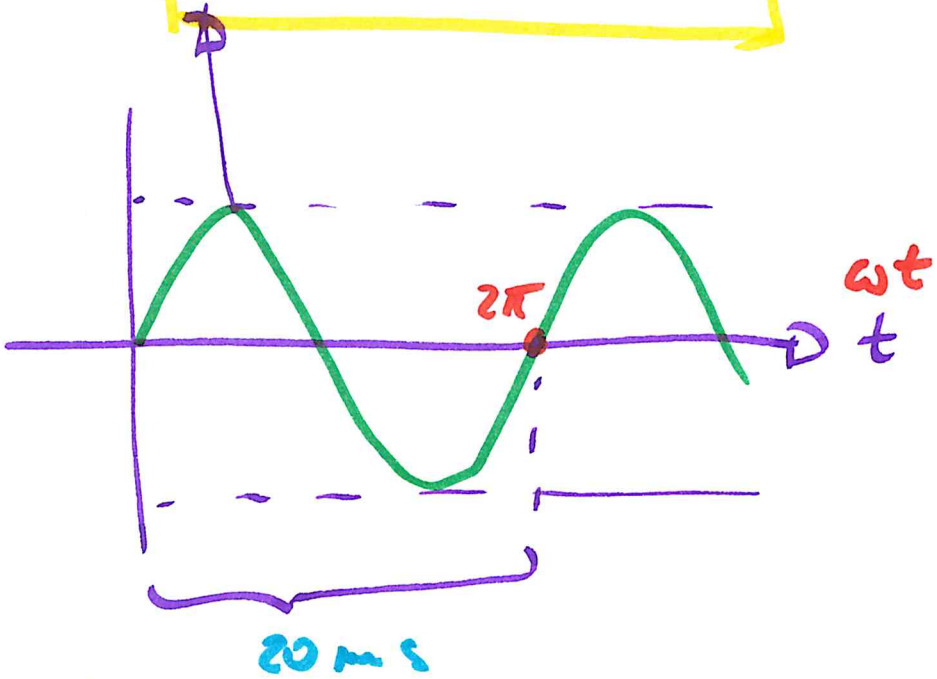
$= u_0 \cdot \sin(\omega t) = u$

$\omega = 2\pi f$ Frequence

⇒ each generator produces the voltage:

see appendix 2.2

$$u = \hat{u} \cdot \sin(\omega t)$$



if $f = 50\,Hz$; $T = \frac{1}{f}$

$T_{50} = 0.02\,s = 20\,ms$

$T_{60} = 0.017\,s = 17\,ms$

• Link between time dep. fuction and complex functions

see appendix 1.2

$$u = \hat{u} \cdot \sin(\omega t) = \hat{u}\ \mathrm{Im}\{e^{j\omega t}\}$$

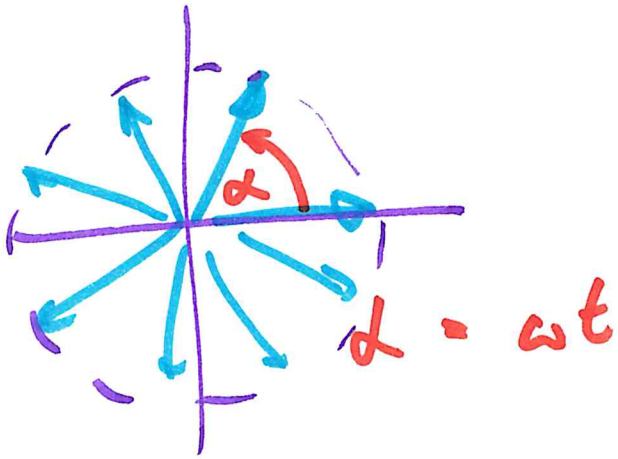
$$= \mathrm{Im}\{\hat{u} \cdot e^{j(\omega t)}\}$$

in general

$$u = \hat{u} \cdot \sin(\omega t + \varphi_u)$$

$$= \mathrm{Im}\{\hat{u} \cdot e^{j(\omega t + \varphi_u)}\}$$

What is: $e^{j\omega}$ $e^{j\alpha} = \underline{z}$



$e^{j\alpha} = e^{j\omega t}$

turning arrow

see appendix 1.2

$$u = \mathrm{Im}\{\hat{u} \cdot e^{j(\omega t + \varphi_u)}\}$$

$$i = \mathrm{Im}\{\hat{i} \cdot e^{j(\omega t + \varphi_i)}\}$$

→ Transformation

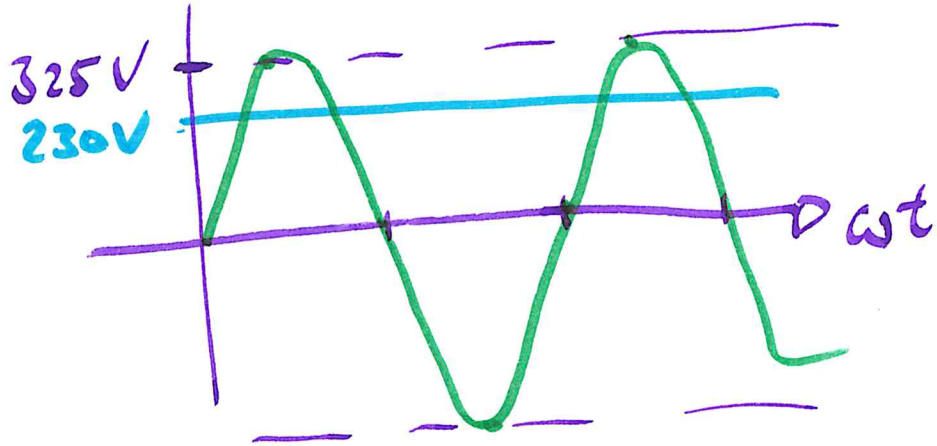
$\hat{u} = \sqrt{2}\,\hat{U}$; $u = \sqrt{2}\,U_{eff}$

$= \sqrt{2}\,U$

$$u = \mathrm{Im}\{\sqrt{2}\hat{U} \cdot e^{j(\omega t + \varphi_u)}\}$$

$$i = \mathrm{Im}\{\sqrt{2}\hat{I} \cdot e^{j(\omega t + \varphi_i)}\}$$

Example: $\hat{U} = 230\,V$



→ addition

(13)

$$u = \mathrm{Im}\left\{\sqrt{2}\,\underline{U}\cdot \underbrace{e^{j(\omega t+\varphi_u)}}_{e^{j\varphi_u}\cdot e^{j\omega t}}\right\}$$

$$= \mathrm{Im}\left\{\sqrt{2}\,\underbrace{U e^{j\varphi_u}}_{\downarrow \underline{U}}\cdot e^{j\omega t}\right\}$$

$$\boxed{\begin{aligned} u &= \mathrm{Im}\left\{\sqrt{2}\,\underline{U}\,e^{j\omega t}\right\} \\ i &= \mathrm{Im}\left\{\sqrt{2}\,\underline{I}\,e^{j\omega t}\right\} \end{aligned}}$$

all complex fuchions have the same turning arrow $e^{j\omega t}$

$$\left\{ a^{m+n} = a^m \cdot a^n \right.$$

(14)

$$u_8 = \mathrm{Im}\left\{\sqrt{2}\,\underline{U}_8\, e^{j\omega t}\right\}$$

$$i_{zt} = \mathrm{Im}\left\{\sqrt{2}\,\underline{\underline{I}}_{zt}\, e^{j\omega t}\right\}$$

Appendix 1.1

$j = \sqrt{-1}$; $j^2 = -1$; $j^3 = -j$; $j^4 = 1$

$\underline{z} = x + j\,y = r\,e^{j\varphi}$; Grafik hierzu mit $\underline{z}(x,y)$, $\underline{z}(r,\varphi)$, $\underline{z}_1$, $\underline{z}_2$ einfügen, kartesische Form – Eulerf.

$r = \sqrt{x^2 + y^2}$; $\varphi = \arctan(y/x)$; der Quadrant ist zu beachten

$x = r\cos(\varphi)$; $y = r\sin(\varphi)$

Wenn $\underline{z}_1 = x_1 + j\,y_1$ und $\underline{z}_2 = x_2 + j\,y_2$, dann ist $\underline{z}_1 \pm \underline{z}_2 = x_1 \pm x_2 + j\,(y_1 \pm y_2)$

Wenn $\underline{z}_1 = r_1 e^{j\varphi_1}$ und $\underline{z}_2 = r_2 e^{j\varphi_2}$, dann ist $\underline{z}_1\,\underline{z}_2 = r_1 r_2\, e^{j(\varphi_1+\varphi_2)}$ mit $\varphi_{1/2} = \arctan(y_{1/2}/x_{1/2})$

Appendix 1.2

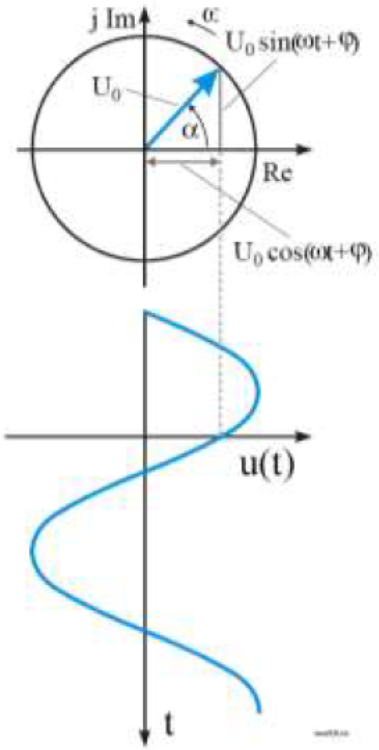


Bild 1 Zeigerdiagramm und Zeitverlauf

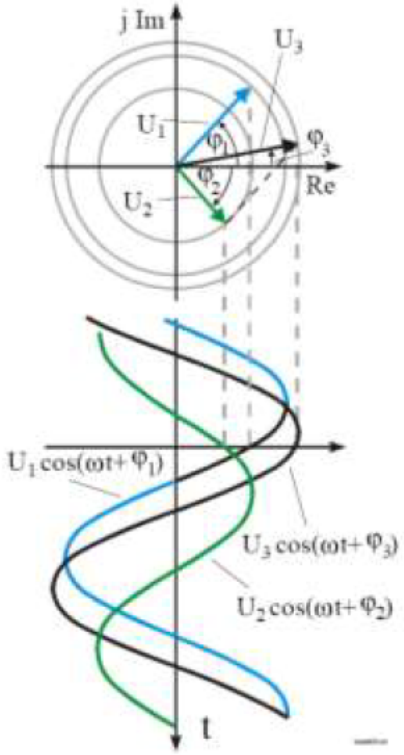


Bild 2 Drei unterschiedliche Verläufe

Hinweis auf: $u(t) = \hat{u}\cos(\omega t) = \mathrm{Re}\left\{\hat{\underline{u}}\,e^{j\omega t}\right\}$

Appendix 2.1

$$\Phi(\alpha) = \Phi(t) = \hat{\Phi}\cos(\alpha) = \hat{\Phi}\cos(\omega\, t) \qquad (1)$$

wenn unter

$\hat{\Phi} = B \cdot A$ der magnetische Fluss als Produkt aus magnetischer Flussdichte und Drahtschleifenfläche gemäß Bild 1a) und unter

$\alpha = \omega\, t$ der Winkel in rad zwischen der Magnetfeldrichtung und der Flächenrichtung (Winkel zwischen dem Flächen- und einem Flusspfeil) als Produkt aus Winkelgeschwindigkeit und Zeit und unter

$\omega = 2\pi f$ die Winkelgeschwindigkeit mit der Frequenz f und beides in $Hz = 1/s$.

$= \frac{2\pi}{T}$ oder mit der Periodendauer T in s

verstanden wird.

Wie bereits im Kapitel 4 angegeben werden für zeitabhängige Größen zumindest bei Strom, Spannung und Leistung kleine Buchstaben verwendet, z.B. $U(t) = u(t) = u$, ansonsten wird die Zeitabhängigkeit angegeben $\Phi(t)$.

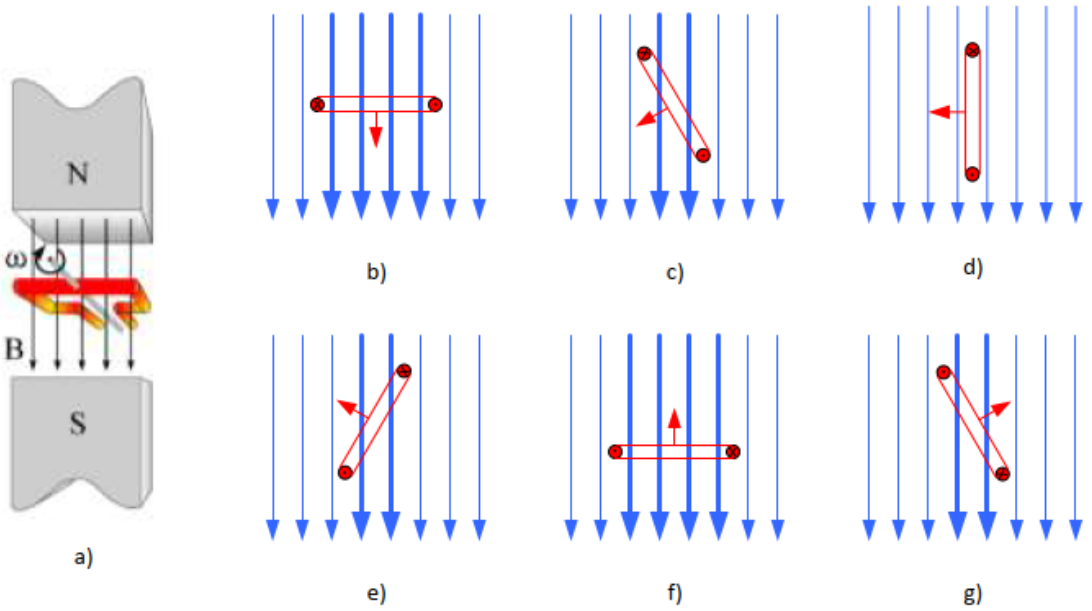


Bild 3 Eine Drahtwindung im homogenen Magnetfeld

Das folgende Bild zeigt das Prinzip eines realitätsnäheren Wechselstromgenerators.

Appendix 2.2

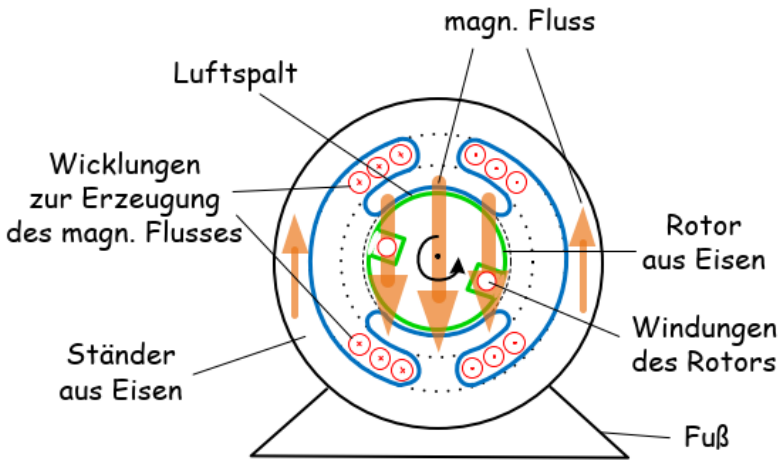


Bild 7 Realitätsnaher Generatoraufbau