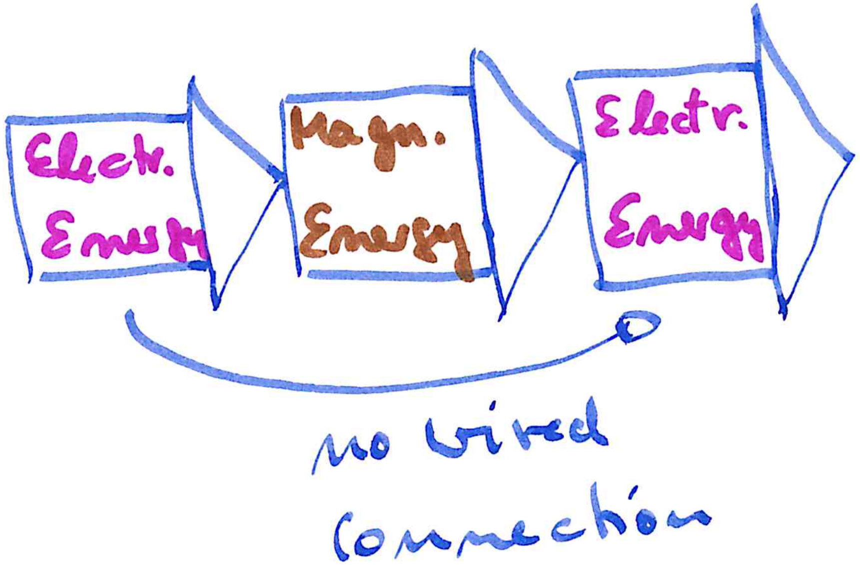
8th January 2020, PeterRue ①

Electrical Machinery

The Transformer (Tr.)



⇒ our high standard of living depends high on electrical energy everywhere

②

⇒ electrical power supply depends high on transformers. Electrical power supply without transformers is not possible (I cannot imagine)

⇒ Picture Voltage levels

⇒ Voltage levels (in Ge…

380 kV	Highest vol…
110 kV	High voltag…
20 kV	Middle volt.
230/400 V	Low voltage

$\Downarrow$

This is not possible without transformers

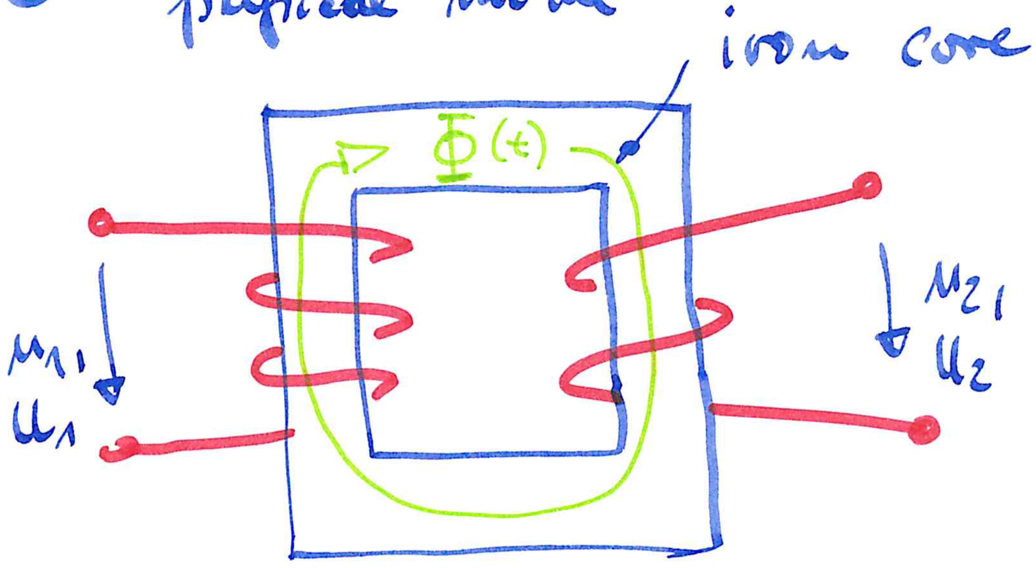
Transformers → ideal Tr. (1.)
→ real Tr. (2.)

- ideal transformer
  - ≙ no losses
  - ≙ $\mu_r \to \infty$
  - ≙ no magnetic leakage flux

symbols:

4 wire symb. 4 w.s. 2 w.s.

physical model



coil with N_1 wind. coil with N_2 winds

we suppose, that inside the iron core is a magnetic flux $\Phi(t) \neq const$ ("God made the flux")

If there is magnetic flux, then at the coils are voltages with the induction laws:

$$u_1 = N_1 \cdot \frac{d\Phi_{\cancel{1}}}{dt}$$

$$=$$

$$u_2 = N_2 \cdot \frac{d\Phi_{\cancel{2}}}{dt}$$

$$\frac{u_1}{N_1} = \frac{d\Phi}{dt} = \frac{u_2}{N_2} ; \quad \frac{u_1}{N_1} = \frac{u_2}{N_2}$$

$$(1) \quad \boxed{\frac{U_1}{U_2} = \frac{N_1}{N_2} = ü = t}$$

Voltage transforming law of an ideal Tr.

t : transmission ratio

ideal Tr. ⇒ no losses ⇒ $P_1 = P_2$ ⇒

$$U_1 \cdot I_1 = U_2 I_2$$

$$\frac{I_2}{I_1} = \frac{U_1}{U_2} = t \qquad (2) \quad \boxed{\frac{I_2}{I_1} = \frac{N_1}{N_2} = t}$$

Current transforming law of an ideal Tr.

$$(1) * (2) \Rightarrow \frac{U_1}{U_2} \cdot \frac{I_2}{I_1} = t^2$$

$$\frac{U_1 / I_1}{U_2 / I_2} = \boxed{\frac{Z_1}{Z_2} = t^2} \qquad (3)$$

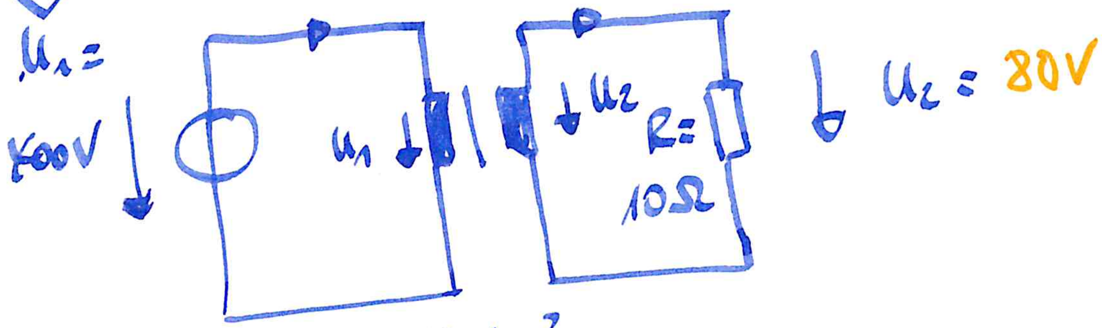
Impedance transmission of an ideal Tr.

7

Exemples

1

$I_1 = ? = 1.6\,A \quad I_2 = ? = 8\,A$



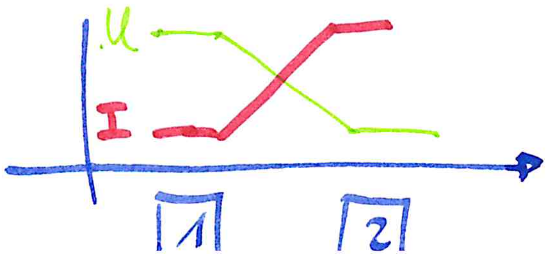
$10 : 2$

$\Rightarrow t = 5$

$$\frac{U_1}{U_2} = \frac{N_1}{N_2} = 5 \; ; \; \underline{U_2} = \frac{U_1}{5} = \underline{\underline{80\,V}}$$

$$I_2 = \frac{U_2}{R} = \frac{80\,V}{10\,\Omega} = 8\,A$$

$$\frac{I_2}{I_1} = t = 5 \; ; \; I_1 = \frac{I_2}{5} = 1.6\,A$$



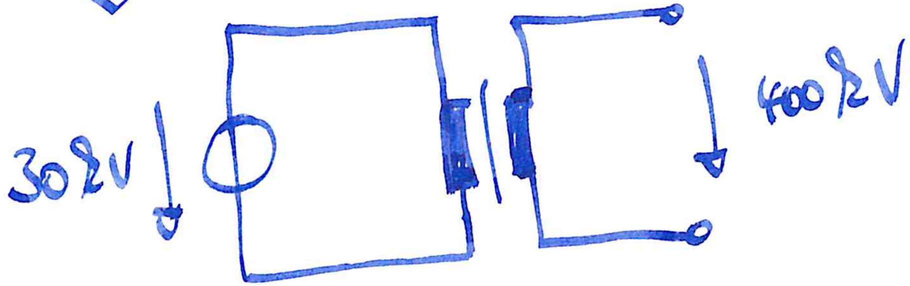
8

$$P_1 = U_1 I_1 = 640\,W$$

$$P_2 = U_2 I_2 = \cancel{10\,W} = 640\,W$$

=

2



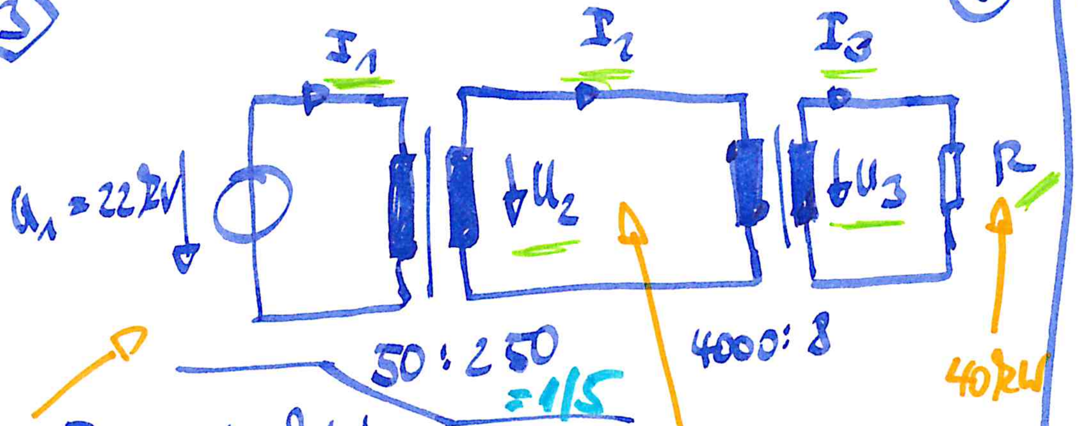
$500 : N_2 = ?$

$$\frac{U_1}{U_2} = \frac{N_1}{N_2} \; ; \; N_2 = N_1 \frac{U_2}{U_1} = \frac{400\,kV}{30\,kV}$$

$$N_1 \cdot \frac{40}{3} = 13.33$$

$$= 6666.7$$

3



$P_R = 40\,kW$

(bei ~~$U_3 = 220V$~~)

$$I_1 = \frac{P_1}{U_1} = \frac{40\,\cancel{k}W}{22\,\cancel{k}V} = 1.8\,A$$

$$\frac{U_1}{U_2} = \frac{N_1}{N_2} = \frac{50}{250} = \frac{1}{5}\,;\quad U_2 = U_1 \cdot 5 = 110\,kV$$

$$\frac{I_2}{I_1} = \frac{1}{5}\,;\quad I_2 = \frac{I_1}{5} = 0.36\,A$$

$$I_2 \cdot U_2 = P_2 = P\,;\quad I_2 = \frac{P_2}{U_2} = \frac{40\,\cancel{k}VA}{110\,\cancel{k}V} = 0.36\,A$$

$$\frac{I_3}{I_2} = \frac{4000}{8} = 500\,;\quad I_3 = 0.36\,A \cdot 500$$

$$I_3 = 180\,A$$

$$P_3 = P = 40\,kW = U_3 \cdot I_3$$

$$U_3 = \frac{40\,kW}{180\,A} = 222\,V$$

*) $P_R = P_3 = 40\,kW$ bei $220V$

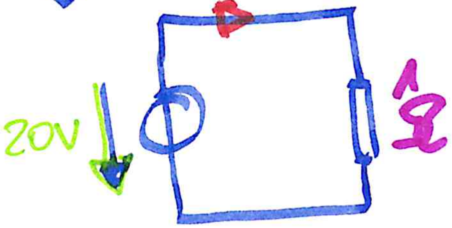
$$P = U \cdot I = I^2 \cdot R = \frac{U^2}{R}$$

$$\Rightarrow R = \frac{U^2}{P} = \frac{(220V)^2}{40\,kW} = 1.21\,\Omega$$

$$P(222V) = \frac{(222V)^2}{1.21\,\Omega} =$$

4

11



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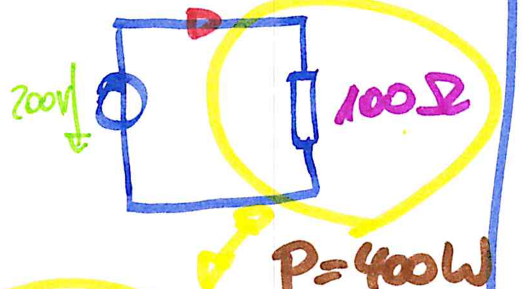

P = 400W

P = 400W

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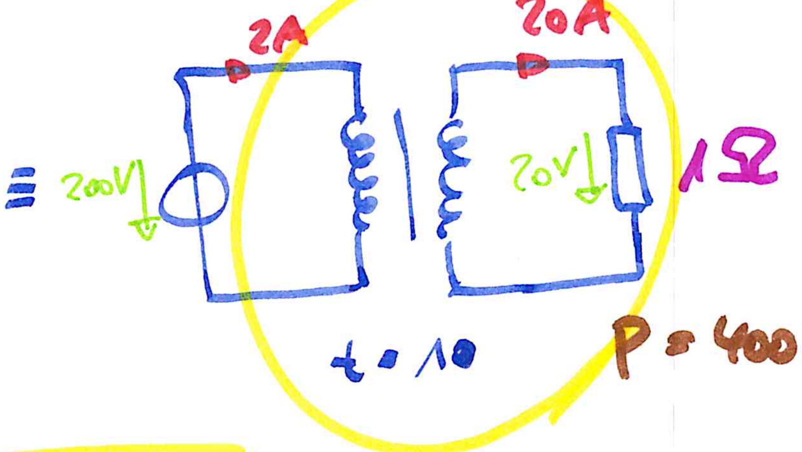


P = 400W

$$\boxed{\frac{Z_1}{Z_2} = t^2}$$

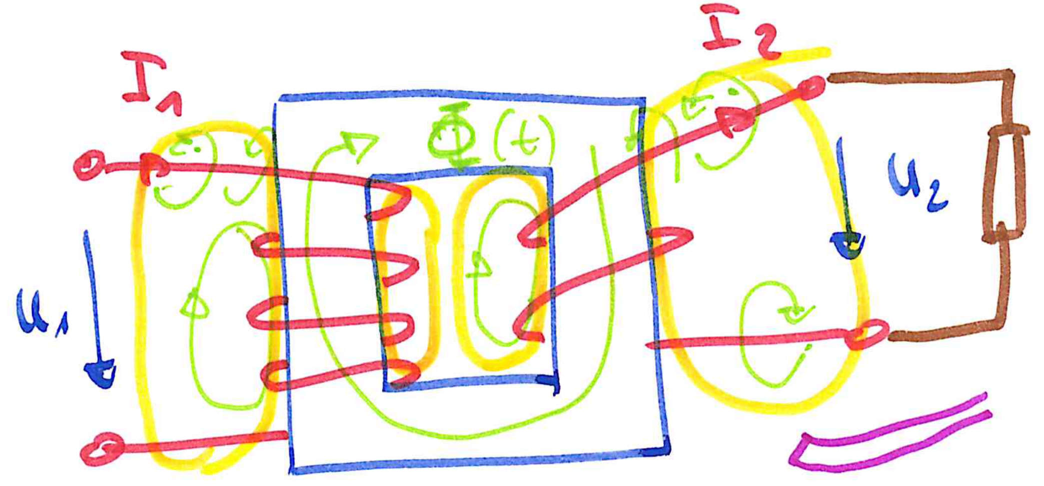
$$Z_1 = Z_2 \cdot t^2 = 1\Omega \cdot 10^2 = 100\Omega$$

Example for transforming impedances

12

- real transformer
  - has losses ($P = I^2 \cdot R$)
  - has hysteresis 
  - leakage flux
  - eddy currents →

flux: $\Phi = L \cdot I$

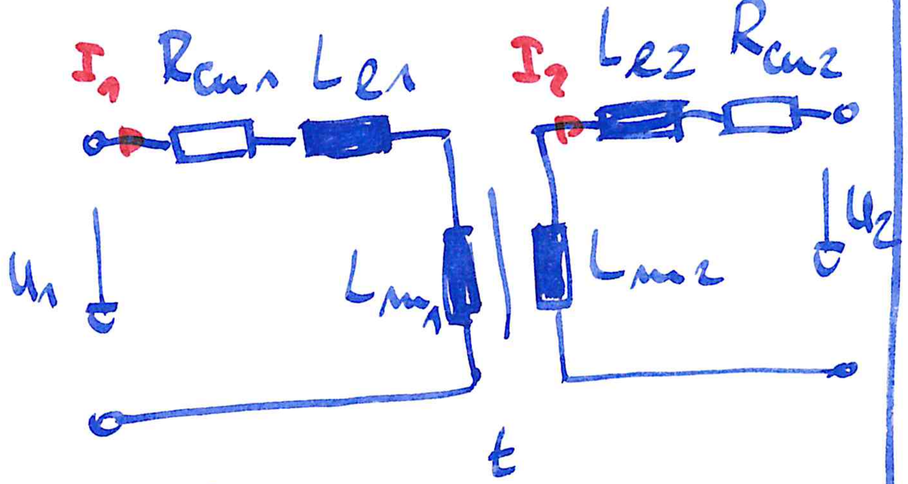


O leakage flux Φ_e

$$\boxed{L_e = \frac{\Phi_e}{I}}$$: leakage inductance

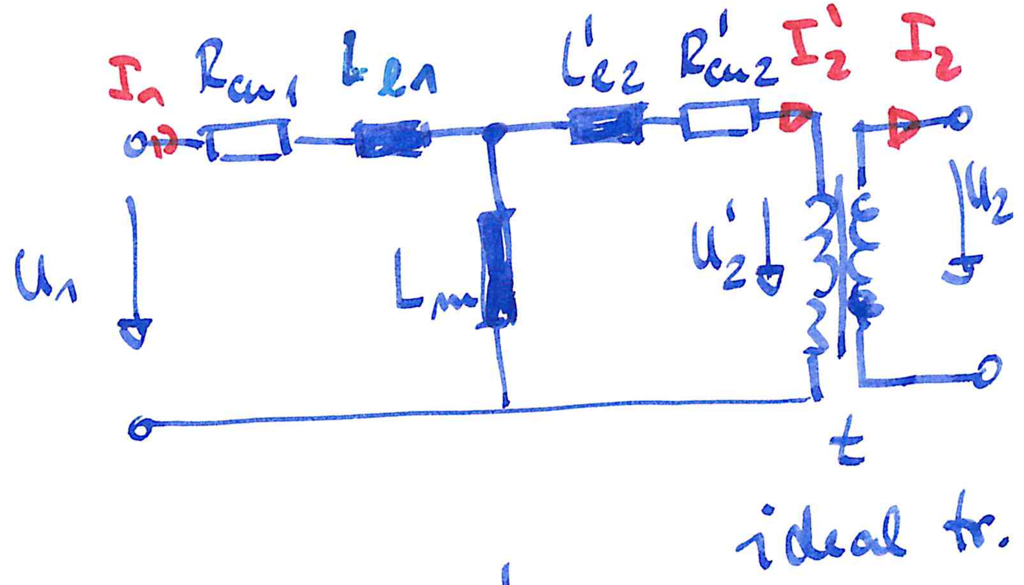
Φ in iron $\rightarrow$ $\Phi_m = \Phi_{main}$

$$\boxed{L_m = \frac{\Phi_m}{I}}$$: main inductance



↳ electric sub-circuit

we bring L_{e2} and R_{cu2} (L_{m2}) to side no. 1 :



$$\frac{I_2}{I_2'} = t \quad ; \quad \frac{U_2'}{U_2} = t$$

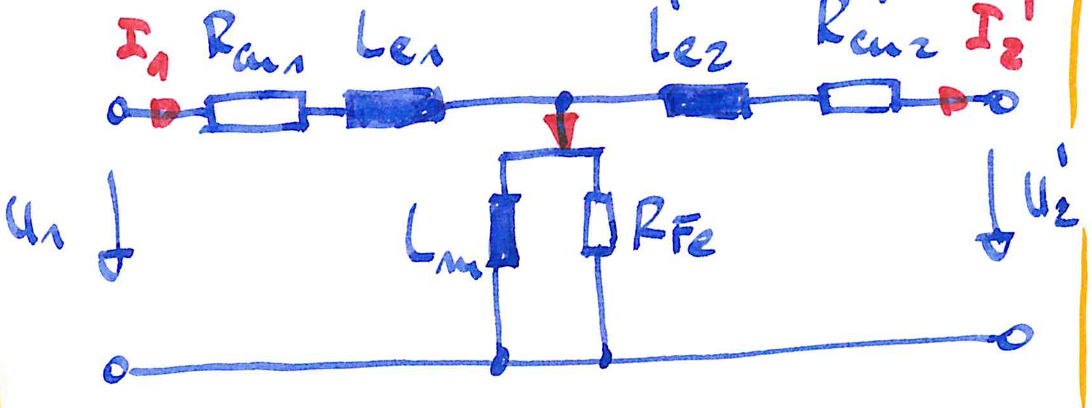
$$I_2' = \frac{I_2}{t} \quad ; \quad U_2' = U_2 \cdot t$$

$$L_{e2}' = L_{e2} \cdot t^2$$

$$R_{cu2}' = R_{cu2} \cdot t^2$$

15

full electric subcircuit for a transformer:



R_{Fe} : R of iron → hyst. losses
→ eddy current losses

$R_{cu1}, R'_{cu2} \ll R_{Fe}$

$L_{\sigma 1}, L'_{\sigma 2} \ll L_m$

16

→ ideal tr.: $R_{cu1} = R'_{cu2} = 0$

$L_{\sigma 1} = L'_{\sigma 2} = 0$

$L_m \to \infty$

$R_{Fe} \to \infty$

$Z_m = \omega \cdot L_m \to \infty$

full electric subcircuit for a ideal transformer:

17

$\Rightarrow I_2' = I_1 = \frac{I_2}{t} \Rightarrow \frac{I_2}{I_1} = t$

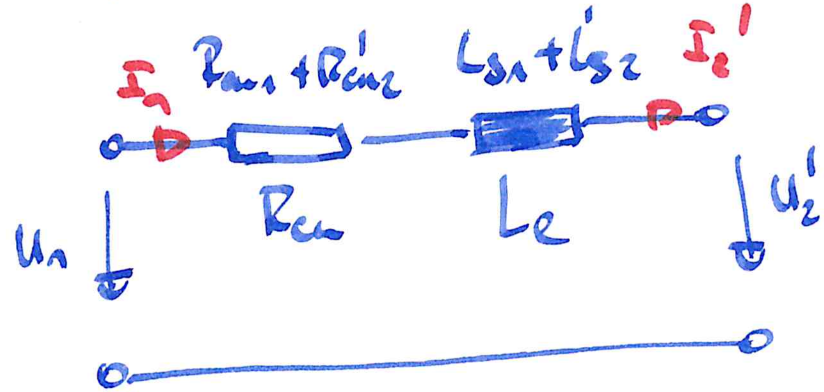
current law of an ideal tr.

$\Rightarrow U_1 = U_2' = U_2 \, t \Rightarrow \frac{U_1}{U_2} = t$

voltage law of an ideal tr.

18

$\Rightarrow$ a transformer with full load:



$\Rightarrow$ a transformer with no load: